THE EFFECTS OF SUBTRACTION ALGORITHMS AND SHORT TERM MEMORY ON COMPUTATIONAL ACCURACY ON FIFTH GRADE STUDENTS

BY

DANIEL ROBERT PAULSON

A DISSERTATION PRESENTED TO THE GRADUATE
COUNCIL OF THE UNIVERSITY OF FLORIDA IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF EDUCATION

UNIVERSITY OF FLORIDA

1983

ACKNOWLEDGMENTS

In completion of this dissertation, I wish to acknowledge the cooperation, enthusiasm, and hard work of the fifth grade classes at Tavares and Triangle Elementary Schools and their teachers in this study of subtraction. I would also like to express sincere and deep appreciation for the advice and time donated by Dr. Robert Lange for the design and data analysis, Dr. Michael Hynes for introducing me to a fascinating topic, Dr. Robert Algozzine for his advice and knowledge in research design and reporting, and Dr. Lynn Oberlin, under whose supervision this experiment was conducted and reported.

I would like to give special acknowledgment to my wife for her patience and to my grandfather whose dream it was to see me through college.

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Abstract of Dissertation Presented to the Graduate
Council of the University of Florida in Partial Fulfillment
of the Requirements for the Degree of Doctor of Education

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By

Daniel Robert Paulson

August, 1983

Chairman: Dr. Lynn C. Oberlin
Major Department: General Teacher Education

This study investigated the effectiveness of three subtraction algorithms as a short duration remedial intervention for multi-digit subtraction computation with students demonstrating high and short term memory aptitudes. From a sample of 121 fifth grade students, sixty-six scored below 80% on a twenty item subtraction inventory and were selected as the subjects for this study. These students were also classified as either high or low memory based upon their performance on the Visual Aural Digit Span Test utilizing the median score of 24 or higher as the indicating high short term memory. Group homogeneity was measured by the control variables of subtraction accuracy on the pretest, IQ as measured by the Otis-Lennon Mental Abilities Test and general computational skill as measured by the

Computation subtest on the Stanford Achievement Test. In separate analyses of variance with each of these variables, the three treatment groups were determined to be nonsignificantly ($p > .05$) different.

Each treatment consisted of supplemental instruction in one of the three subtraction algorithms (decomposition, equal additions, and Hutchings' low stress). The treatments were composed of a series fifteen lessons of approximately twenty minutes each taught by the regular classroom teacher.

As a measure of treatment effectiveness, a subtraction posttest consisting of twenty multi-digit subtraction problems was administered following the treatment completion. The analysis of variance indicated a significant ($p < .05$) difference between treatment posttest means with a significant ($p < .05$) difference between the equal additions mean (8.566) and the decomposition treatment mean (12.177). The low stress treatment mean (7.000) was also significantly different from the decomposition mean, but not from the equal additions treatment mean. No significant aptitude-treatment interactions were found between treatments. Error analysis of the posttest performances indicated a larger incidence of errors associated with borrowing which involves zeroes than any other type of error. It was concluded that as a short term remedial intervention, alternative algorithms do not appear to be as effective as a very sequential review of the traditional algorithm.

CHAPTER I

INTRODUCTION

This study investigated the effects of instruction in three subtraction algorithms on computational accuracy. The possibility of an aptitude-treatment interaction was investigated between students demonstrating low short term memory and the three subtraction algorithms.

Statement of the Problem

While the elementary school mathematics curriculum has become considerably broader than just the mastery of the basic operations, there has been an increased demand from society for higher computational skill levels. In an effort to develop a better understanding of mathematics and better problem solving skills, the elementary school mathematics curriculum has been broadened to include many areas which were once reserved for the secondary curriculum. Yet, mastery of the basic arithmetic operations is still the primary goal of the elementary school curriculum. For many students, this crowded curriculum has not devoted enough time for total mastery of the basic computational procedures. There has been a considerable body of research dealing with the difficulties students experience in learning computational skills, especially the patterns and types of errors made by students in the four basic operations of addition, subtraction, multiplication and division. This research has indicated that students exhibit more errors in the subtraction algorithm than any of the other three basic operations (Cox, 1975). Bright (1978) has found that subtraction with regrouping can be more difficult than multi-digit multiplication.

Remediations aimed at correcting computational errors range from repetitive drill to interventions aimed at facilitating cognitive development. Alternative algorithms have been prescribed as a fresh and new remedial strategy which may "circumvent the mind set of failure which beleaguers the child" (Ashlock, 1976, p. 11). Still, very little research has been conducted with alternative algorithms concerning their efficiency at remediating computational problems. It is apparent that the most efficient and effective method of developing these skills is needed if students are to attain a high standard of computational proficiency which is being demanded by society. For students having difficulty learning these operations, a more efficient and effective method of performing arithmetic computation would be of significant value.

Significance of the Problem

In recent years there has developed a popular and professional concern over the computational competence of school children. This popular issue is a part of the "back to the basics" movement in mathematics education. In part, this movement is a result of numerous complex political, economic, psychological and educational forces. It also has roots in the culmination of over a decade of intense activity in mathematics curriculum development.

In the period of time beginning with the launching of Sputnik in 1957 until 1970, there was a well financed effort to accelerate scientific and mathematical achievement in American education. The mathematics curricula which were generated by this activity came to be known as the "New Math." Hill (1979) summarized the essence of the changes which were recommended for elementary school mathematics education as follows: use the language of mathematics; introduce new content such as informal geometry; place a greater emphasis on understanding and rationalizing techniques, rather than on rote learning skills; use the structure of the subject matter as an organizational principle.

New instructional methods were also recommended. The use of guided discovery, inductive and deductive models, active participation by the learner and the use of concrete or manipulative aids were advanced as more psychologically sound methods of instruction. The ideas of Piaget and Bruner became very prominent in the literature. Hill (1979) notes that the concept of the "spiral curriculum" has been responsible for introduction of content at earlier levels in school where computational drill once prevailed. As a result, computational skills have suffered (Kane, 1973).

A longitudinal study of computational abilities in New Hampshire found that scores for eighth graders in arithmetic computational skills were decreasing (Austin & Prevost, 1972). It was noted that most teachers appear to be placing "more emphasis upon the understanding of arithmetic, hence less time is spent on drill and practice in computation" (pp. 63-64). This decrease was noted across groups receiving "new math" transition and traditional mathematics instruction. Tenth graders who experienced the "new math" curriculum made significantly higher scores in mathematics achievement than the traditional group. It was hypothesized that the greater emphasis upon algebra and geometry and problem solving skills in the test and the curriculum rather than computational skills accounted for this effect.

Another study utilized a meta-analysis technique to study the effectiveness of the new math curriculum (Athappilly, 1978). This study summarized 134 studies on new mathematics located in the literature. When the meta-analysis technique was conducted on these data it was found that new mathematics curricula did improve academic achievement and attitude toward mathematics. These improvements were most notable in algebra. Those approaches which emphasized concrete presentation of abstract concepts were identified as more effective in this study.

While the mathematics curriculum has expanded, the time children spend on mathematics has remained the same. The use of concrete to abstract sequencing has facilitated concept development in mathematics, however, this conceptual knowledge has taken up time used to develop computational mechanics. Hutchings (1976, p 219) states, "Arguments that the new curriculum develops better understanding of the operations are beside the point. Understanding division is not the same as knowing how to divide quickly and accurately." Despite this revolution in mathematics curriculum there has not been a sufficient amount of research and development of more efficient abstract computation algorithms. This study was undertaken as an attempt to test the effectiveness of two alternative subtraction algorithms and the standard subtraction algorithm in developing computational accuracy in subtraction.

Limitations and Assumptions

This investigation was limited to six fifth grade classes in the Central Florida area. Only three subtraction algorithms were tested with limited teacher training in the procedures. Random assignment of students to treatment groups was not possible, but all of the students were randomly assigned to their respective classrooms at the beginning of the school year.

The following assumptions were required with the design utilized in this study (Myers, 1956, p. 61):

1. The differences in the subjects are independently distributed.
2. The individual differences are normally distributed.
3. The variance of the individual differences is the same for all treatment groups.

Definition of Terms

Abstract--Symbolic representation of real things and actions. Numbers and arithmetic signs are abstract representations.

Algorithm--A systematic procedure for finding the result of an operation on two numbers when the result is not apparent (Jungst, 1975, p.79).

Concrete--The manipulation and use of real objects rather than using paper and pencil abstractions.

Decomposition Subtraction Algorithm--A subtraction algorithm in which borrowing is accomplished by adding ten to the digit being subtracted and by decreasing the next digit to the left by one. This is the traditional subtraction algorithm taught in North American elementary schools.

Equal Additions Subtraction Algorithm--A subtraction algorithm in which addition is used to simplify the borrowing procedure. For example:

$$\begin{array}{rcrcr} 435 & + & 43 & = & 478 \\ -\underline{157} & + & 43 & = & -\underline{200} \end{array}$$

Low Stress Algorithm--A set of procedures for the arithmetic operations which are characterized as having a unique subscript notation and discrete operations.

Minuend--The upper or first number in a subtraction problem from which another number is subtracted.

Short Term Memory--The temporary storage mechanism which serves to hold either newly acquired information until it is processed for long term storage or to hold previously acquired information retreived from long-term memory storage for use in the present (Webster, 1978).

Subtrahend--The number which is being subtracted from the minuend.

Superscript--A notation of reduced size for the recording of intermediate operations and the place values in Low Stress procedures.

Hypotheses

The following null hypotheses were tested:

H_1 There is no significant ($p < .05$) difference in computational accuracy in subtraction for fifth grade students after receiving instruction in either decomposition, equal additions or low stress subtraction algorithms.

H_2 There is no significant ($p < .05$) difference in computational accuracy in subtraction for fifth grade students with low short term memory after receiving instruction in either the decomposition, equal additions, or the low stress subtraction algorithm.

H_3 There is no significant ($p < .05$) difference in computational accuracy in subtraction for fifth grade students with high and low short term memory after receiving instruction in decomposition subtraction.

H_4 There is no significant ($p < .05$) difference in computational accuracy in subtraction for fifth grade students with high and low short term memory after receiving instruction in the equal additions algorithm.

H_5 There is no significant ($p < .05$) difference in computational accuracy in subtraction for fifth grade students with high and low short term memory after receiving instruction in the low stress algorithm.

Summary

There has been an increased demand for computational accuracy in elementary school students by society in the last few years. The literature indicates that subtraction with regrouping is a very difficult operation and one that is not mastered until the upper elementary grades.
The elementary school mathematics curriculum has expanded greatly in the last two decades such that teachers and students do not have adequate time to teach difficult subtraction with regrouping. Students with learning difficulties are particularily handicapped by the curriculum at this level. The use of alternative algorithms has been recommended as a remedial method for students who are experiencing difficulty mastering the standard computational algorithms. Yet, the research which has been conducted in

this area has not investigated the effectiveness of these algorithms as remedial procedures with students demonstrating a lack of mastery of subtraction and particularily with students with diminished short term memory capacity.

The purpose of this study is to investigate the effectiveness of three methods of subtraction in relationship with short term memory with fifth grade students. A method which produces computational accuracy in a short period of time for students with short term memory deficits would be valuable asset to the mathematics curriculum.

CHAPTER II

REVIEW OF THE LITERATURE

Subtraction is the inverse operation of addition. Jungst (1975, p. 20) defines subtraction as "For all $a, b \in W$; $a \geqslant b$, the difference $a - b$ is the unique whole number c such that $a = c + b$." Piaget and Szeminska (1952) found that in order to understand the concept of reversibility children have to have developed certain cognitive structures characteristic of the concrete-operational stage. Before the child reaches this stage, he cannot reverse his thinking. He cannot relate the end to the beginning. At the concrete operations stage a cognitive structure is developed which frees the child from the here and now. The child can relate operations to concrete-empirical experiences both current and recent. The operations that are developed include classification, ordering and operations of a spatial and temporal nature. Two forms of a reversibility operation develop: "Negations, in which a perceived change can be seen to be cancelled by a corresponding, negating operation; and an elementary form of the reciprocal relationship, in which, for example, 'being a brother' is seen as implying that there is another sibling in the family who reciprocates the relationship" (Collis, 1975, p.26). Basically, negation annuls tha operation directly while reciprocation neutralizes the operation without stopping its execution.

Piaget studied the conceptual development of children's mathematical development. In his work, he identified many important notions of number and classification underlying this development. Computational skills are logically dependent upon these concepts for their structure. Still, the development of basic computational skills in children are not dependent upon the development of the logical structure upon which they are built. Collis (1975, p. 260) states, "a child can memorize both sequence of number names associated with counting and the elementary addition and subtraction facts without comprehending the basic conepts underlying them."

Subtraction can be thought of in three different models (Reisman, 1977, p. 231). Take-away subtraction is the most elementary form. It involves taking away a part of a set, which leaves a reduced number of elements. Problems of this kind are of the following sort: "John had six marbles. He gave two to Jim. How many marbles did he have left?" (6 - 2 = _). Problems in this model are either giving or taking away part of a set.

The second model of subtraction is comparison type subtraction. This model involves comparing two numbers in order to find the difference between them or the number of the smaller set when the number of one set and the difference is given. Problems in this model are as such: "Sam has three gerbils; Roy has one. How many more gerbils does Sam have than Roy?"

The third type of subtraction conceptualizes the operation as decomposition of a number into its component parts. Reisman (1977) considers this algorithm appropriate for third grade and up. Conceptually it involves finding the number of elements in the one subset when the number of elements in the other subset and the number of elements in the combined set are known. It can also involve dividing a set into two subsets. Problems of this type are "John has walked three miles on a five mile journey. How many miles does he have left to walk?" Activities which involve separating a set several different ways are also illustrative of this type of subtraction.

These three types of subtraction are appropriate for basic fact operations but are not useful for larger numbers. For larger numbers more abstract algorithms are necessary which can manipulate the place value system. Jungst (1975) states,

> It certainly is true that we could use the definition for a basic operation in the whole numbers and counting as a method for performing a numerical

> calculation. However, it is not very practical to perform a numerical calculation using only the basic operation definition and counting with numbers whose numerals have two or more digits" (p. 79).

Reisman (1977) identifies four abstract algorithms for subtraction. These procedures facilitate subtraction computation but may not by themselves facilitate understanding of the operation.

The first two procedures are forms of the "equal-additions method" (Reisman, 1977, p. 248). They are procedures which add to both the minuend and the subtrahend to either simplify the problem or allow for easier borrowing.

The first method is the complementary method. It is based upon the additive-identity of 10, whose inverses sum to zero. This method replaces digits in the subtrahend with as many zeros as possible, creating an easier problem. For example,

$$\begin{array}{rcr} 541 + 7 & = & 548 \\ -193 + 7 & = & \underline{-200} \\ & & 348 \end{array}$$

The second algorithm is called "borrowing and repaying plan." This procedure borrows from the minuend by adding to the next digit in the subtrahend. For example,

$$\begin{array}{rl} 7234 & \text{8 from 14, 6; 10 from 13, 3;} \\ \underline{-3798} & \text{8 from 12, 4; 4 from 7, 3} \\ 3436 & \end{array}$$

The third method is called the "decomposition take-away" method of subtraction. This procedure is sometimes called simple borrowing. Sherrill (1979) states that the decomposition algorithm is the most popular in North America. It consists of borrowing one ten from the next digit and adding it to the digit in the minuend being subtracted from. For example,

143
-87 7 and what make 13, 6; 8 and what make
56 13, 5.

The clerks method of determining how much change to return is also an additive technique (Wilson, 1951, p. 145). The clerk will usually add coinage amounts to the sales total up to the amount tendered. For example, on a $7.34 purchase with a $10 bill tendered the clerk will,

add	$ .01	to make	$ 7.35
add	.05	to make	7.40
add	.10	to make	7.50
add	.25	to make	7.75
add	.25	to make	8.00
add	1.00	to make	9.00
add	1.00	to make	10.00
total	2.66		

Reisman (1977, p. 249) states that the decomposition method is an easy method to demonstrate at the concrete level. Transfer from the concrete to the symbolic should be logical and consistant. Jungst (1975,) states that "children should be able to formulate an algorithm after they have solved many numerical problems using manipulative devices" (p. 80). Manipulative devices may be any article that can be bound together in groups 1's and 10's to demonstrate the place value system. For example, popsicle sticks, tongue depressers, plastic straws, centimeter rods, or the open ended abacus could be used to demonstrate the decomposition method of subtraction.

Research on Subtraction

There is an extensive body of research on addition and subtraction. Romberg (1982) identifies two studies that have had a profound influence on mathematics education: Thorndike's instructional suggestions in his Psychology of Arithmetic (1922) and Brownell's studies on subtraction. While Thorndike provided an instructional model from which arithmetic was taught for several decades, Brownell (1947) and Brownell and Moser (1949) profoundly influenced the computational algorithms taught in most American schools. Their classic studies examined the equal additions and decomposition algorithms under two instructional methods, a rationale method and a mechanical method. The rationale method involved instruction in both the rationale for the procedure and the mechanics of the algorithm. The mechanical method involved instruction only in the procedure used by the algorithm. The Brownell study compared four treatment groups with each other with the following results:

(1) The decomposition rationale treatment was more effective than the decomposition mechanical for both applications and computational accuracy.

(2) The equal addends rationale treatment was more effective than the equal addends mechanical for applications.

(3) Both equal addends treatments were more effective than the decomposition mechanical in both applications and computational accuracy.

(4) the decomposition rationale treatment was more effective than either addends treatment in all areas.

Suydam and Weaver (1975, p.8) state that even though the decomposition algorithm has been used exclusively, the equal addends algorithm can also be taught meaningfully. "With inceased emphasis in many programs on properties an on the compensation in particular, equal addition method can also be presented with meaning. For instance, pupils are learning that:

(a) $9 - 3 = \_\_\_$ means that

$\_\_\_ + 3 = 9$ or $3 + \_\_\_ = 9$

They are learning that:

(b) $7 - 4 = 3$ is equivalent to

$(7 - 4) + 2 = 3 + 2$

Development of such ideas should facilitate the teaching of equal additions procedure" (Suydam and Weaver, 1975, p. 8).

Sherrill (1979) investigated the effectiveness of the decomposition and the equal additions algorithms with two randomly assigned groups of twenty-four third graders. After a four week treatment, it was found that the decomposition group performed significantly better on a test for computational accuracy. An analysis of the errors made by both groups revealed that the students in the equal additions treatment group produced more errors in subtracting the minuend from the subtrahend as well as more uninterpretable errors than the students in the decomposition treatment.

Brown and VanLehn (1982) reported on a system for diagnosing systematic computational errors students make which has precisely defined the bugs found in place-value subtraction. The term "bugs" was taken from computer science in which it means that some erroneous variation of procedure is being used in the execution of a task. A student's subtraction errors are considered to be some erroneous variation in the subtraction algorithm caused by either faulty knowledge or memory failure. Inaccurate performances are not viewed as resulting from a lack of skill knowledge, but rather as variations in the skill procedure which are invented by the student and are substituted for the correct procedure.

In an exhaustive analysis of childrens' subtraction, Brown and Burton (1978) observed eighty-eight primitive bugs in the standard subtraction algorithm. These bugs are capable of occurring singularly or as compound bugs. The total number of bugs which are logically possible is approximately 3000, of which 300 were reported as having been observed. The three most common bugs are smaller -from-larger, in which the subtrahend and minuend are interchanged; borrow-from-xero, in which the zero is changed to nine, but no decrement is made in any column left of it; and 0 - N = N, where the result is the same as if the procedure was addition. The following are a sample other bugs which were identified (Brown & VanLehn, 1982, p. 121):

Don't-Decrement-Zero

When borrowing across a 0, the student changes the 0 to 10 instead of 9.

Stops-Borrow-At-Zero

Instead of borrowing across a 0, the student adds 10 to the active column but doesn't decrease the column to the left.

Zero-Instead-of-Borrow-From-Zero

The student won't borrow if he or she has to borrow across 0. Instead, the student will write 0 as the answer to the column requiring the borrow.

The bugs theory hypothesizes that when students attempt a task with impoverished procedures, the student will attempt to repair the algorithm with his or her own invented procedures developed from his or her problem solving skills. These attempts at repairing the incomplete algorithm generate perturbations in the procedure which are the characteristic bugs. Brown and VanLehn (1982, p. 130) identified four major repair rules which account for at least twenty known bugs. These are

I. Escape and Flee
    a. Skip
    b. Quit
    c. Back up to last choice
II. Change the operation slightly
    a. Switch arguements of the same data type
    b. Refocus left
    c. Refocus down

III. Use an operation that worked in an analogous situation (e.g. Increment for decrement)

IV. Dememorize

These repair rules are considered to be instances of higher order general purpose heuristics which the student applies in any problem solving situation.

Certain tests and signals are learned which alert the student that certain repair rules are not correct. These are called critics (Brown & VanLehn, 1982, p. 131). Critics alert the student that something is not working the way it should. These signals are generally ignored when bugs are present. Critics are said to be "tacitly acquired by the students' observing and abstracting the patterns that all computations appear to satisfy--especially those that were produced by the teacher working through example solutions to homework problems" (Brown & VanLehn, 1982, p.131). Critics are abstracted into three general categories:

Critics Based on the Form of the Writing

1. Don't leave a blank in the middle of the answer.
2. Don't have more than one digit per column in the answer.
3. Don't decrement a digit that is the result of a decrement.

Critics Based on Information Theoretic Properties

1. Don't change a column after its answer is written (or more generally, each operation must make a difference to the answer).
2. Don't borrow twice for the same larger from smaller column (or more generally, avoid infinite loops).
3. Both digits of the column ahould affect its answer (or more generally, there is no superfluous information in a subtraction problem.

Critics Based on Semantics

1. Unless borrowing is involved, a column's answer must not be greater than the top digit.
2. Subtraction is not commutative, so don't reverse the column.

Resnick (1982) analyzes the written subtraction algorithm as a set of syntatic rules coupled with a set of semantic rules. Syntactic rules describe the mechanics of the procedure and when strictly followed can solve any multi-digit subtraction problem which is written in aligned column format. Performing the algorithm requires no understanding of the base system and the reasoning behind the operations are omitted. The semantic rules are those properties of the base system and the logic upon which the operations in the algorithm are based upon.

Drawing from the theory of bugs (Brown & Burton, 1978), Resnick postulated that bugs occur in the written subtraction algorithm because children are unable to use their semantic knowledge to support written syntatic procedures. Using a case study design with four children, Resnick (1982) followed over the course of a year the development of addition and subtraction skills and the relationship with semantic knowledge of the algorithm. Interviews were conducted in November, February, and May in which the children were asked to perform addition and subtraction problems and demonstrate understanding of the semantics of the base system and the procedures. The students were asked to represent written numerals in concrete forms and to add and subtract using concrete representations. Particular attention was given to the correspondence between the written and the concrete representations.

It was found that three of the four children had acquired a strong knowledge of the base system in concrete representation at the beginning of the study. One child was acquiring knowledge of the system during the cource of the year. By May, all of the children could perform subtraction with borrowing with concrete objects. Resnick noted that "because there is little practice in the children's school curriculum on subtraction using these concrete representations, it seems reasonable to suppose that the much more skillful and semantically correct performance they all demonstrated by May was not the result of practice on subtraction routine itself, but of the development of general semantic knowledge of the base system and subtraction" (1982, p. 145).

Examination of the children's written work found that they seemed to have knowledge of the base system, but that this knowledge was more syntactic than semantic. When attempting to write numbers in expanded notation the children used a trial and error method and demonstrated difficulty with numbers containing zeros. Analysis of the children's work with the written algorithm revealed the development of three key components: left-to-right rules of procedure, the carry operation, and the borrow procedure. None of the zero bugs were observed with these four children.

Resnick (1982) noted that there appeared to be a "lack of correlation between knowledge of the base-system semantics and knowledge of the written algorithms" (p. 148). The link between these two systems is weak with the result being that children who have difficulty remembering the written procedure can not draw effectly upon the semantic knowledge to generate appropriate procedures. It was suggested that "initial instruction that stresses the semantic properties of addition and subtraction algorithms should help block the difficulties and buggy foutines that might arise later (Resnick, 1982, p. 149).

The research on subtraction reported here has shown that with a little exception, the standard algorithm has been unchallenged since Brownell's studies in the 1940's. The subtraction algorithm appears to be a rather complex procedure given that there are over 3000 possible bugs which can theoretically occur during its execution. The most difficult portion of the algorithm is in dealing with zeroes during the regrouping procedure. The research has also indicated that a semantic understanding of the number system may not be necessary to perform the abstract algorithm but that without such knowledge, the student must depend entirely upon his memory of the mechanics of the procedure to succesfully perform the calculation. It has been theorized that an inadequate knowledge of the number system coupled with an impoverished memory can result in computatioanl procedures with bugs.

The Low Stress Algorithm

In other reviews of the literature on computational algorithms, Hutchings (1972), Dashiell (1974), and Lester (1978) reported very little research upon any new algorithms. These researchers have called for a change in the structure and teaching of computational skills. A set of experimental algorithms has been introduced by Hutchings (1976) called "Low-Stress Algorithms". Hutchings feels that the low stress algorithms may provide an effective alternative to the traditional structure and teaching of mathematics. Hutchings states, "They appear to permit easy mastery after a brief training, to provide greater computational power than conventional algorithms, to operate with much less stress on the user than conventional algorithms, and to enjoy certain other advantages" (Hutchings, 1976, p. 219). These techniques cover the whole number operations of addition, subtraction, multiplication, and division. Their application, it is recommended, should be directed towards remediations with upper elementary and junior high students.

A low stress algorithm is distinguishable from a conventional algorithm by two unique characteristics (Hutchings, 1975, p. 227):

(1) A concise, definable, easily read, supplementary notation is used to record every step.

(2) The learner can complete any intermediate step of a distinct kind rather than alternate between different kinds of intermediate steps.

These two characteristics focus attention on binary operations in only one mode at a time consequentially eliminating shifting from multiplcation to addition continuously in a multiplication problem. In addition and subtraction a complete record of fact recall is made separate from the regrouping operation. The predicted outcome with these procedures is that there will be a reduced demand on memory and imaginary manipulation compared to the conventional algorithm. A second outcome will be a reduction in dissonance and retroactive inhibition derived from the alternation of operations in conventional algorithms (Hutchings, 1976, p. 237-238).

The low stress subtraction algorithm offers a good example of these characteristics. This algorithm uses a format and procedure which is characterized by four features: notation, minuend position, renaming sequence and digits renamed. First, a superscript is used to denote place value. A half sized "1" is placed at the upper left of a numeral indicating the overloading of that place in regrouping. For example, 435 could become $3^{1}35$ or $42^{1}5$. The minuend is positioned such that there is a space between it and the subtrahend in which the renamed minuend is written. The renamed minuend is not separated from the subtrahend by the crossed out minuend in this procedure as it is in the conventional procedure. This convention helps some children in reading and organizing their work. A low stress subtraction problem appears as such:

$$\begin{array}{r} 7\ 6\ 9\ 4\ 2 \\ \\ -\underline{3\ 7\ 7\ 9\ 2} \end{array}$$

The procedure begins by regrouping the entire minuend before any subtraction occurs. The sequence appears as the following:

$$\begin{array}{rrrrrr} & 7 & 6 & 9 & 4 & 2 \\ & & & & {}^{1}4 & 2 \\ - & 3 & 7 & 7 & 9 & 2 \\ \hline \end{array} \qquad \begin{array}{rrrrrr} & 7 & 6 & 9 & 4 & 2 \\ & & & 8 & {}^{1}4 & 2 \\ - & 3 & 7 & 7 & 9 & 2 \\ \hline \end{array} \qquad \begin{array}{rrrrrr} & 7 & 6 & 9 & 4 & 2 \\ & & {}^{1}6 & 8 & {}^{1}4 & 2 \\ - & 3 & 7 & 7 & 9 & 2 \\ \hline \end{array} \qquad \begin{array}{rrrrrr} & 7 & 6 & 9 & 4 & 2 \\ & 6 & {}^{1}6 & 8 & {}^{1}4 & 2 \\ - & 3 & 7 & 7 & 9 & 2 \\ \hline & 3 & 9 & 1 & 5 & 0 \end{array}$$

After all the regrouping is completed the subtraction is completed in one uninterupted sequence. The renaming procedure is reported to provide a simplified process that is also demonstratable at a concrete level (Hutchings, 1975, p. 229). This procedure can be easily demonstrated using manipulatives much the same as the standard algorithm is demonstrated.

When regrouping across zeroes is encountered, the student simply ignores the zeroes and skips over to the number that allows borrowing. The regrouping operation continues with temporary empty spaces beneath the zeroes until the all of the nonzero columns are completed. The final task in the regrouping process is to simply fill in any blank spaces with nines. This procedure is illustrated as such:

$$\begin{array}{rrrrrrr} & 5 & 2 & 0 & 0 & 0 & 1 \\ & & & & & & {}^{1}1 \\ - & 2 & 8 & 7 & 6 & 5 & 4 \\ \hline \end{array} \quad \begin{array}{rrrrrrr} & 5 & 2 & 0 & 0 & 0 & 1 \\ & & 1 & & & & {}^{1}1 \\ - & 2 & 8 & 7 & 6 & 5 & 4 \\ \hline \end{array} \quad \begin{array}{rrrrrrr} & 5 & 2 & 0 & 0 & 0 & 1 \\ & 4 & {}^{1}1 & & & & {}^{1}1 \\ - & 2 & 8 & 7 & 6 & 5 & 4 \\ \hline \end{array} \quad \begin{array}{rrrrrrr} & 5 & 2 & 0 & 0 & 0 & 1 \\ & 4 & {}^{1}1 & 9 & 9 & 9 & {}^{1}1 \\ - & 2 & 8 & 7 & 6 & 5 & 4 \\ \hline & 2 & 3 & 2 & 3 & 4 & 7 \end{array}$$

If there is no regrouping across the zeroes, the student does not follow this procedure. The zero is handled as any other number which is not regrouped. This is illustrated as

$$\begin{array}{r} 8\;0\;9\;0\;7\;0\;6\;0\;0 \\ 8\;0\;9\;0\;7\;0\;6\;0\;0 \\ \underline{-5\;0\;4\;0\;6\;0\;6\;0\;0} \\ 3\;0\;5\;0\;1\;0\;0\;0\;0 \end{array}$$

This procedure requires the student to discriminate between zeroes which are skipped over and changed to nines.

Lester (1978) studied the effectiveness of low stress addition and subtraction with educationally disadvantaged rural elementary children. One hundred forty-nine first, second and third grade children were given instruction in basic addition and subtraction using either Hutchings' low stress treatment or the conventional treatment. The third grade children were given instruction in subtraction facts, column addition, and row subtraction. It was found that the low stress treatment was more effective at the .01 level of significance than the conventional treatment in promoting computational accuracy.

Boyle (1973) indentified one advantage with the low stress algorithm as the reduced amount of information stored in short term memory. This is accomplished by converting much of the information generated by intermediate steps to written notation and by avoiding the switching back and forth from one operation to another. Hutchings (1972) has postulated that conventional algorithms utilize all six of Guilford's (1967) memory abilities as compared to one memory ability (memory for isolated items of information) needed for low stress algorithms.

Kalin and Shami (1981) investigated the effectiveness of the low stress procedures in addition and subtracation in comparison to the traditional algorithm for additon and subtraction. Mildly intellectually handicapped students in one school system were assigned randomly to a treatment and a control group for two and one-half hours of instruction in each algorithm. The classroom teachers conducted the instruction in the experimental and the traditional algorithm after an inservice activity in which the teachers were trained in the use of the algorithm and presented a demonstration lesson by a consultant.

The resuts indicated a significant ($p<.001$) mean gain difference between experimental and control groups in both addition and subtraction. The percentage increase in problems solved correctly between pretest and posttest was also very dramatic for the elementary students in the low stress subtraction group. It was reported that this group increased their subtraction competence by 3188% (i.e. from a mean number correct of 0.16 to a mean of 5.10).

Hutchings has presented an alternative computational set of procedures called low stress algorithms. The low stress subtraction algorithm appears to offer an interesting remedial procedure. It attempts to reduce the demands placed upon short term memory through a concise notation and through the performance of each operation in its entirety before progressing to the next step in the procedure. The research which has been reported to date indicates that in a short term remedial intervention with mildly intellectually handicapped students, and in an extended intervention with the algorithm presented as the primary procedure, the low stress subtraction did demonstrate effectiveness in increasing subtraction accuracy. None of the studies reported data on the difficulty of the subtraction problems over which these gains were achieved.

The Role of Short Term Memory In Mathematics

The role of short term memory in mathematical ability was investigated by Collis (1975). From the cognitive development viewpoint, an invididual's short term memory would be directly related to the level of reasoning, concrete or formal, of which the individual would be capable (p. 129). It was deduced that a memory span of four would be necessary to perform at the level of concrete operations. To perform at the level of formal operations, a memory span of six would be required. These were meant to be minimum requirements which appear to be necessary for the ability to develop strategies characteristic of the concrete-operational stage and the formal-operational stage.

Collis (1975) compared the operational level of reasoning to both age and memory span. A cross-sectional sample of 330 children was administered a digit span test and six operational tests. It was found that digit memory span is related to levels of cognitive development and the ability to solve mathematical problems characateristic of concrete operational thinking and formal-operational thinking.

In another study, Webster (1978) investigated the differences in short term memory for one group of mathematicly proficient and two groups of mathematicly disabled sixth graders. Three groups of six graders were selected in a stratified random manner on the basis of performance in the arithmetic subtest of the Wide Range Achievement Test (WRAT). On the basis of this measure they were assigned to a control group, a mildly disabled group, and a severely disabled group.

Memory recall was measured using serial lists of digits and non-rhyming consonants. Each string was presented both aurally and visually during an individual testing session.

The data were analyzed for each group by the modality presentation (aurally or visually) matrix with intelligence as a covariate. The results revealed that mathematically proficient students performed significantly better than either of the mathematically disabled groups. Webster (1978) postulated that average achievers are employing different coding strategies during short term recall than those used by poor achievers. Proficient students performed better in both aural and visual presentation modes with more information recalled when presented aurally. The mathematically disabled students recalled more information with the visual presentation.

Koppitz (1977) investigated the relationship of short term memory to achievement using the Visual Aural Digit Span test (VADS). The VADS consists of four subtests which interrelate visual and aural input with written and oral output modes to yield eleven measures of intersensory and intrasensory processing and a global short term memory score. The entire fifth grade (N=26) of a middle class community school was administered the VADS and the comprehensive Test of Basic Skills (CTBS). It was found that seven of the eleven VADS measures were significantly related to the total arithmetic measure of the CTBS.

Hamsin (1978) reported on an investigation of the information processing models for addition, subtraction, and multiplication in mentally retarded and non-retarded children. It was found that non-retarded fourth graders utilized a memory-retrieval system for processing subtraction problems. Retarded children were more prone to use a counting algorithm to solve basic subtraction problems.

Case (1982) has suggested that along with experience, the development of central processing capacity plays an important role in cognitive development. It was concluded that the development of central processing capacity may be determined by two factors. Case (1982, p. 162) identifies these as,

> First, the basic factor that controls the growth of processing capacity is operational efficiency. In effect, a child's total processing capacity may be thought of as fixed, and the amount available for storage may be seen as a function of the proportion for this total capacity that must be devoted to executing basic arithmetic operations. A second conclusion is that the operational efficiency is a function of both maturation and practice. Stated differently, practice can improve a child's operational efficiency in a given domain, but only up to the limit imposed by maturation.

In mathematics, children exhibit the same pattern of development as other abilities (Case, 1982, p. 156). That is, children may be seen to operate on levels of relational complexities. In a stage-like manner, children develop more capacities to enable them to handle greater quantitative loads that complex procedures demand. Case (1978) identifies short term memory as a critical factor in cognitive processing and development. He suggests that "young children tend to develop reasonable but oversimplified strategies for coping with tasks that are set at school, and that one reason they do so is that they are incapable of coping with the informational demands of the learning situation" (Case, 1978, p. 342). The instructional implications from this theory imply that instruction should be at the same relational complexity at which the student currently functions or slightly above that stage. The load placed upon short term memory should be minimized as well as ensuring that the child's operations are as efficient as possible.

Summary

The decomposition subtraction algorithm has been the standard algorithm utilized in abstract computation for several decades. Brownell, in a classic study, demonstrated that the decomposition algorithm with rationale instruction was superior to both decomposition under a mechanical approach or to equal additions in either the rationale or the mechanical instuction. Recently, the decomposition algorithm again demonstrated superiority over the equal additions algorithm despite the heavy emphasis that has been placed upon the structure of the number system by the new mathematics curricula.

Research on mathematics and cognitive abilities has identified short term memory as a major factor in determining mathematics achievement. It has been shown that the ability to understand and to perform arithmetic computation and solve problems is dependent upon the capacity of the individual to store and retrieve information. Individuals with impaired or diminished short term memory capacity have shown lower achievement scores in arithmetic. Any method or procedure which would reduce the demands placed upon short term memory or which would facilitate more efficient use of memory and information processing should reduce the effect of memory deficits upon arithmetic achievement scores.

Hutchings has developed a new series of algorithms which appear to hold promise for accelerating the computational accuracy of upper grade elementary students. The low stress subtraction procedure has demonstrated effectiveness in dramatically increasing the subtraction skill of mildly intellectually handicapped students after a brief intervention. It also has demonstrated significant gains over instuction in the traditional subtraction when it becomes the major algorithm presented to disadvantaged third grade students along with the addition algorithm in an extented intervention. None of the studies reported the complexities of the problems mastered nor the types of errors exhibited by students after instruction in low stress subtraction. There appear to be several unanswered questions in the literature:

1. What is the effectiveness of a short intervention with the low stress algorithm with upper elementary students who have demonstrated the need for remedial interventions in subtraction.
2. What types of errors are characteristic of students after a short intervention with low stress subtraction.
3. Can a short intervention in the decomposition, low stress, or equal additions algorithm remediate errors in multi-digit subtraction with regrouping across zeroes.

CHAPTER III

METHOD

This study employed a mixed, three-way hierarchical nested design (Myers, 1966; Keppel 1973; Afifi & Azen, 1979) which consisted of random treatments nesting classrooms with fixed short term memory categories. Six fifth grade classes in two schools were randomly assigned to three remedial treatments such that each school had one class in each of the three treatments. Within each classroom, students were assigned to either a high or low short term memory category. Group homogeneity was measured by the control variables of subtraction inventory, IQ, and the Stanford Achievement Test arithmetic computation subtest scores. For this study, a priori mastery of subtraction skill was set at 80% correct on the inventory (Terwilliger 1971, p. 98). Students demonstrating less than 80% mastery of the subtraction algorithm were determined to be in need of remedial instruction and selected for data analysis. A subtraction posttest was administered to all subjects at the end of the treatment as a measure of treatment effectiveness for students demonstrating lack of mastery in subtraction skills. An analysis of variance procedure was used as a test for the effects on the following set of factors: treatments, groups, and memory levels. Treatment effects and aptitude-treatment interactions were tested using an analysis of variance procedure for nested designs.

Procedure

Each treatment consisted of supplemental instruction in subtraction given during the regular mathematics class by the regular classroom teacher. The fifteen lessons had a duration of approximately twenty minutes. Each teacher was given instruction in each technique and the material to be used for each lesson. Each teacher was observed during the treatment period to ensure that the technique was understood by the teacher and students. The treatment duration lasted three weeks. Data were gathered before and after the treatments.

Each lesson consisted of the teacher demonstrating the skill to be taught on the chalk board or overhead projector. Each lesson also had a worksheet which provided instructions on the procedure to be used and several practice problems to be completed by the student. The teachers were free to help individual students as was needed. The worksheets were collected after each lesson.

The two teachers in each treatment group received individual training in the algorithm which was to be taught by them in their classrooms. They did not receive any instruction in the other algorithms until after the treatments ended. The inservice design consisted of (1) training designed to increase the competence of teachers in the use of one algorithm; (2) a demonstration providing an opportunity for the teachers to observe a consultant teaching students the algorithm; and (3) a follow-up question-answer session.

Treatment one used the conventional decomposition algorithm with a highly sequential skill progression. The lessons consisted of a demonstration, group practice, and individual work. The class was given a worksheet with the appropriate problems for individual work. The teacher brought the lesson to a close with the collection of the work sheets. These were analyzed by the experimenter for accuracy and the number of problems attempted.

The lessons in treatments one and two were designed in a skills hierarchy. Gagne (1977) used subtraction of whole numbers as an example in his discussion of the "learning task analysis" (p. 107). This analysis attempts to identify the prerequisite skills necessary to perform the target objective. For subtraction of whole numbers, the prerequisite skills are identified as (p. 107),

(a) subtracting single digits in a row with no borrowing

(b) subtracting with borrowing from the adjacent column

(c) subtracting with successive borrowing in adjacent columns, and

(d) subtracting with borrowing across zeroes.

The lessons controlled the subtraction facts and algorithm complexities demanded from the student. The final objective was subtraction of six-digit numbers with all possible fact combinations and with borrowing across multiple zeroes.

Treatment two consisted of instruction in using the equal additions algorithm over the same skill sequence as treatment one. The lesson format was the same as treatment one. The major difference between treatment one and two was the algorithm the students used.

Treatment three consisted of instruction in the low stress algorithm. As suggested by Hutchings (1976), the treatment was designed in three stages: (a) problems in which all columns require borrowing with no zeroes, (b) problems in which all columns require borrowing with some borrowing over zeroes, and (c) problems in which intermittent borrowing includes borrowing across zeroes.

Sample

The subjects in this study were the students in six fifth grade classes in two elementary schools. Three classes were randomly assigned a subtraction algorithm to be taught in each school. The schools are located in an agricultural region in Central Florida with enrollments of approximately 500 students each in grades K-5. Both schools have a wide socioeconomic range with approximately 12% of the students in each school qualifing for Chapter 1 services and approximately 30% of the students representing a wide range of minorities. The total sample in all six classes contained 121 subjects with a mean of 13.724 on the twenty item subtraction inventory instrument and a standard deviation of 4.138. From this sample all those demonstrating questionable mastery by scoring less than 80% correct (Terwilliger, 1971, p. 98) on the subtraction inventory were selected for further analysis. This remedial group consisted of 66 subjects with a mean score of 10.419 and a standard deviation of 4.248. The means and standard deviations for the control variables of subtraction inventory, SAT computational scores, and IQ are presented in Table 3-1 for the remedial group.

Table 3-1

Means For Subtraction Inventory, Short Term Memory, Stanford Achievement Test Computational Grade Level Scores, IQ for Each Treatment Group, Classroom, and Memory Group

	N	SUBT INV	MEM	SAT	IQ
Low Stress Treatment					
CLASS 1	11	9.1	22.3	5.08	99.6
High Memory	5	11.4	24.4	4.98	106.6
Low Memory	6	9.2	20.5	5.18	92.6
CLASS 4	15	10.4	24.4	5.85	99.6
High Memory	10	10.2	25.8	6.10	102.3
Low Memory	5	10.7	22.4	5.50	95.8
Equal Additions Treatment					
CLASS 2	5	14.4	24.4	5.96	103.8
High Memory	2	16.0	27.5	7.00	112.5
Low Memory	3	13.7	22.3	5.26	98.0
CLASS 5	18	8.7	23.9	5.21	100.0
High Memory	8	8.1	26.1	5.10	106.9
Low Memory	10	9.3	21.7	5.30	93.2
Decomposition Treatment					
CLASS 3	7	11.0	21.8	5.06	96.8
High Memory	2	10.5	25.5	6.30	102.5
Low Memory	5	11.6	20.4	5.80	94.6
CLASS 6	10	12.6	23.5	5.90	100.4
High Memory	5	11.6	26.0	5.80	99.6
Low Memory	5	13.6	21.0	6.00	101.2

Note. Total Possible 20.0 28.0 grade level

Employing a nested model of analysis, prior differences and group homogeneity were tested to ensure that any differences between treatment groups at the onset of the treatment were minimal (Cochran, 1957; Winer, 1962). This test consisted of a separate analysis of variance for each of the control variables (subtraction inventory score, IQ, and SAT computation score). Data from the subtraction inventory, computational subtest of the Stanford Achievement Test, and Otis Lennon IQ test were analyzed as dependent variables for the test of group homogeneity. Analysis of variance was computed by the SAS procedure General Linear Models with the subtraction inventory scores as the dependent variable to be analyzed by treatment class, classrooms within treatments, memory groups, memory groups within each treatment, and memory group within each class room within each treatment ordered in a nested hierarchical model. The results from this analysis are presented in Appendix A. The analysis of variance for the subtraction inventory indicated nonsignificant F values for memory classifications, memory classifications within treatments, and memory classification within classrooms which permitted pooling of mean square terms for these nested groups. A pooled mean square of 17.055 with 61 degrees of freedom was obtained by pooling the nested sum of squares with the error sum of squares divided by the combined degrees of freedom associated with each term. This pooled mean square was utilized in calculating the F ratio for determining the significance of the differences between the nested classrooms within treatment classes. A nearly significant ($p<.052$) F value of 2.705 with 3 and 61 degrees of freedom was obtained for classrooms nested within treatments. The analysis of the differences in treatment groups in this model produced a nonsignificant ($p>.20$) F value of 0.500 with 2 and 3 degrees of freedom for treatment groups for the subtraction inventory performances of students with less than 80% correct.

An analysis of variance was computed with the computational subtest scores from the Stanford Achievement Test as the dependent variable to be analyzed by treatment class, classrooms within treatments, memory groups, memory groups within each treatment, and memory group within each classroom within each treatment ordered in a nested hierarchical model. The results from this analysis are presented in Appendix B. The Analysis of variance for the computational subtest scores indicated nonsignificant ($p>.20$) F values for memory classifications, memory classifications within treatments, and memory classification within classrooms, classes within treatments, and treatment groups.

The results of the analysis of variance for IQ as the dependent variable are presented in Appendix C. The independent variables of treatment class, classrooms within treatments, memory groups, memory groups within each treatment, and memory group within each classroom for each treatment group were analyzed in a nested hierarchical order. The analysis of variance for IQ scores yielded nonsignificant ($p > .20$) F values for all levels of the analysis except for memory classification which was found to have significant ($p < .05$) differences in IQ scores between low and high memory classifications. There was a significant differance between high and low memory groups on IQ scores.

Instrumentation

A priori subtraction accuracy was measured on an instrument constructed by the examiner (Appendix D). The instrument consisted of 20 six-digit subtraction problems. Two and three strings of zeroes were randomly used in the subtrahend to test the skill of carrying across zeroes. This instrument was constructed to present a variable set of skill demands.

Arithmetic achievement was measured by the calculations subtest of the Stanford Achievement Test (SAT) which was administered by the school system. The Stanford Achievement Test has a reliability of .92 for the fifth grade form. The Arithmetic Computation subtest for the beginning fifth grade form has a mean grade-level equivalant of 5.2 and a standard deviation of 1.2 grade levels. IQ was measured by the Otis Lennon Mental Abilities Test which was also administered by the school system. The Otis-Lennon has a reported reliabilities of .89 on alternate forms and of .91 for test and retest comparison. The validitity of the Otis-Lennon when compared to the SAT Arithmetic Computation subtest is reported as .60 for the fifth grade form.

Short term memory was measured by the Visual Aural Digit Span Test (Koppitz, 1977) (VADS). This instrument with a reported reliability of .90 (Koppitz, 1977) was individually administered by the experimenter. The instrument consists of four tasks: repeating digits heard, repeating digits seen, writing digits heard, and writing digits seen. Performance of these four tasks yields a total test battery score. The Total VADS score has a reported correlation of .45 ($\underline{p}<.001$) with IQ as measured by the WISC (Koppitz, 1978, p. 115). For this study, short term memory was categorized as being either high or low. High short term memory was a score at or above the group median. Low short term memory was a score below the group median.

Posttest subtraction accuracy was measured with a twenty-item subtraction instrument (Appendix E) written by the examiner. The posttest items ranged in difficulty from a three-digit subtraction problem with no borrowing to a twenty-three digit subtraction problem with varied borrowing demands. The posttest was designed to measure the following skills:

(a) Borrowing from adjacent columns

(b) Borrowing to a zero

(c) Borrowing across a zeroes

(d) Discriminating when to borrow to a zero.

Design and Data Analysis

The design of this study employs a mixed hierarchical design in which treatments were nesting classes with fixed high and low short term memory groups. In this study fifth grade students in two schools were randomly assigned to classrooms at the beginning of the school term by the counselors and principals of each school. A classroom in each school was randomly assigned to one of the three treatments. Students in each treatment were also assigned to a high or low memory classification depending upon their performance on a test of short term memory. Myers characterizes this design as "subjects within classes within methods" (1966, p. 212) and identifies the total variability among subjects as having three potential sources:

(a) Treatment effects. The variable A (treatment) is a potential source.

(b) Group effects. Differences in the composition of groups (school classes, schools and memory groups) may contribute to the variability in the data.

(c) Residual individual differences. Even the scores of subjects within the same group may vary due to such factors as attitude or ability.

The assumptions associated with the design of this student are tested under the residual error term in the data analysis.

The independent variables in this study were Treatment group, Classroom and Memory classification. The dependent variable was the subtraction posttest scores. This dependent variable was entered into the data analysis procedure to determine treatment effects and aptitude treatment interactions. Analysis of variance was the statistical procedure utilized for this analysis with the computer program General Linear Models in the SAS statistical package. Means and standard deviations were also computed utilizing the procedure Means from the SAS statistical package. These procedures were executed using the computer services at the Northeast Regional Data Center of the State University System of Florida in Gainesville.

The analysis of variance technique is used to determine if significant differences exist in a number of factors and for the combination of those factors at more than one level or subgroup in a given population (Afifi & Azen, 1979, p.197). The total sample variance, called the Error term or Residual Error, is compared to the individual subgroup variances and the interactions of these group assignments. The Null Hypothesis predicts that there are no differential effects between subpopulations. This is measured by the ratio of the factor variances over the Error variance which forms the statistic called the F ratio. The F ratio is calculated by dividing the mean square of the factor by the mean square of the error term. The error term contains an unbiased estimate of the total variance in the population. In a normal distribution with no differences between the means of the subpopulations, the F ratio equals one. If the means for individual groups vary beyond what would make it unlikely that the differences between the Mean Squares associated with the factor and the total population is by sampling error, then the F value is called significant and the Null Hypothesis is rejected.

The General Linear Models procedure calculates the Sum of Squares (SS) value and the degrees of freedom associated with each factor and the Error term for the model. The Mean Square (MS) for each factor and the total sample is obtained by dividing the Sum of Squares (SS) by the Degrees of Freedom (Df). The F ratio is calculated by dividing the Mean Square by the Mean Square associated with the Error term for that factor.

In the analysis of variance for a hierachical design containing a within-groups variable, the F ratio for each source of variance is not determined using the same mean square. Instead, each source of variance is considered according to the subgroups in the hierarchy from which the source is derived. The sources of variance and the tests for significance for the nested within-groups model are presented in Table 3-2 (Myers 1966, p. 223).

TABLE 3-2
Sources of Variance and the Tests of Significance for the Nested Within Groups Model

A	$\frac{\text{MSa}}{\text{MSg/a}}$
G/A	$\frac{\text{MSg/a}}{\text{MSerror}}$
M	$\frac{\text{MSm}}{\text{MSgb/a}}$
AM	$\frac{\text{MSam}}{\text{MSgm/a}}$
GM/A	$\frac{\text{MSgm/a}}{\text{MSerror}}$

In this study the sources of variance are Treatment, Treatment nesting Classes, Memory, Treatment by Memory, Nested Classes by Memory within Treatments. Classrooms are a nested factor in this analysis. The F values are calculated using the Error terms which provide an unbiased estimate of the total variance for each level in the hierarchy. Utilizing this model of analysis of variance with the subtraction posttest scores as the dependent variable, Null Hypotheses One through Five postulated in Chapter I were tested with the rejection level set at $p<.05$.

Summary

Six fifth grade classes in two Central Florida schools were administered a twenty item subtraction inventory, the Visual Aural Digit Span Test (Koppitz, 1977) and were randomly assigned to one of three treatment groups. The three treatments consisted of fifteen sequential lessons in either the decomposition subtraction algorithm, the equal additions subtraction algorithm, or Hutchings low stress subtraction algorithm. Students in each class were also assigned to either a high or low short term memory classification depending upon the total VADS score. Students with scores above the median were assigned to the high short term memory group. All other students were assigned to the low short term memory group. All students participated in the treatments, but only those demonstrating the need for remediation in subtraction (66) were selected for data analysis. Control variables of SAT computational grade level equivalant, Otis Lennon IQ scores and subtraction inventory scores were utilized in a test for systematic bias. An analysis of variance for these variables indicated that each remedial treatment group was not significantly different ($\underline{p}<.05$) on each control variable at the onset of the treatments.

As a test of treatment effectiveness for remedial interventions, a posttest of twenty subtraction problems ranging in difficulty from three digits with no borrowing to twenty-three digits with varied borrowing was administered. An analysis of variance for nested designs was employed as a test for significant differences between treatment groups on the posttest performance in order to determine differential treatment effects. A hierarchical design also tested for aptitude-treatment effects for students with high or low short term memory in each of the treatment groups. The means and standard deviations for the posttest were also computed. A post hoc analysis of posttest means for simple effects utilizing Scheffe's Multiple Comparisons Method was performed to determine the significance of the differences between treatments and between memory classes.

CHAPTER IV

FINDINGS

In this study, 66 students demonstrating the need for some remedial intervention in subtraction from six fifth grade classes from two schools were assigned to one of three remedial subtraction treatments for a brief fifteen-lesson intervention. The students were also assigned to either a high or low memory classification according to their performance on a test of short term memory. Treatment effects were measured on a twenty item subtraction posttest.

Data Analysis

The data collected in this study were analyzed by the SAS procedures Means and General Linear Models utilizing the computer services at the Northeast Regional Data Center of the State University System of Florida located on the campus of the University of Florida in Gainesville. These procedures were used to test the Null Hypotheses presented in Chapter One for this sample, that is, the extent to which differences in posttest performances can be accounted for by treatment and short term memory classification for a group of fifth grade students demonstrating lack of mastery with subtraction.

The Visual Aural Digit Span Test (Koppitz, 1978) was administered to all of the students in the six classrooms of the study. For 130 fifth grade students in these six classes, the median total recall for all four subtests was 24. For this study, students were assigned to the high memory group if their total VADS score was greater than 23. Students with total VADS scores less than 24 were assigned to the low memory group for data analysis. There were 32 subjects assigned to the high memory group, while the low memory group had 34 subjects assigned to it. In order to assure that each cell in the analysis had more than one member, it was necessary to use a VADS score of 23 or greater for the high memory classification in Class 3 since there was only one subject which met the criteria of having less than 80% on the subtraction inventory and having a VADS score of 24 or greater. Only one student had a VADS score of 23 in this class.

The means and standard deviations for the twenty item posttest are presented in Table 4-1 for each treatment group and classroom. The overall mean was 11.00 with a standard deviation of 5.339. The decomposition treatment achieved the highest mean score on the posttest of the three treatment groups followed by the equal additions and the low

Table 4-1

Means and Standard Deviations for the Posttest

Group	N	Mean	S.D.	Group	N	Mean	S.D.
Low Stress							
Class 1	11	6.6	6.00	Class 4	17	6.9	6.80
High Memory	5	9.8	7.44	High Memory	10	6.8	7.20
Low Memory	6	3.9	1.95	Low Memory	7	7.2	5.84
Equal Additions							
Class 2	5	8.4	5.46	Class 5	20	8.6	5.37
High Memory	2	13.0	3.00	High Memory	10	5.4	1.41
Low Memory	3	5.3	4.49	Low Memory	10	11.2	5.47
Decomposition							
Class 3	7	14.0	4.11	Class 6	10	13.5	4.77
High Memory	2	18.0	2.00	High Memory	5	12.2	4.70
Low Memory	5	12.4	3.38	Low Memory	5	12.8	4.79

stress treatment groups. The results also show that the high memory classified students did not consistently achieve higher mean scores on the posttest within this sample.

An analysis of variance was computed with the posttest scores to test Null Hypothesis One in which a nonsignificant difference in posttest performance after receiving instruction in either of the three treatments was predicted. In this analysis, the dependent variable posttest was analyzed by treatments nesting classrooms and memory classifications in a hierarchical model. The results from this analysis are presented in Table 4-2. A significant ($p<.05$) F value was found for the main treatment effects along with a significant ($p<.05$) F value for the interaction of the nested effect of classrooms by memory classification within treatments. Nonsignificant F values were found for all other sources of variance.

TABLE 4-2

Analysis of Variance for Posttest

Source	Df	ANOVA SS	MS	F	P
Treatment	2	375.991	187.99	11.540	.05
Treatment/class	3	48.868	16.29	0.202	<.20
Memory	1	75.541	75.54	0.937	<.20
Trt*Memory	2	13.779	37.77	0.468	<.20
Class/Memory(Trt)	3	241.110	80.59	2.826	.05
Error	54	1539.775	28.51		

The means and standard deviations for the posttest scores by treatment group and memory classification are presented in Table 4-3. Post hoc analysis of treatment effects using Scheffe's Multiple Comparison method (Afifi and Azen, 1979) was utilized to determine the simple treatment effects This analysis indicated a nonsignificant ($p>.05$) difference between the mean of the low stress treatment (X = 7.0) and the mean of the equal additions treatment (X = 8.6) groups. A significant ($p<.05$) difference was found between the posttest means for the decomposition (X = 12.2) and the equal additions treatments. The difference between the low stress posttest mean and the decomposition posttest mean was also significant ($p<.05$).

TABLE 4-3

Means, Standard Deviations and Mean Differences For the Posttest for Treatments and Memory Classifications

Group	Mean	S.D.	Difference	P
Low Stress	7.000	6.038	1.566	<.20
Equal Additions	8.566	5.173	3.611	.05
Decomposition	12.177	5.604		
High Memory	9.031	6.183	0.060	<.20
Low Memory	8.971	5.723		

Null Hypothesis Two predicts that there will be a nonsignificant ($p>.05$) difference between posttest means for the low memory classified students across treatments. An analysis of the simple effects for each treatment low memory mean was computed utilizing Scheffe's Multiple Comparisons method. A difference of 4.483 between the low stress low memory and the equal additions low memory means was found to be significant at the .05 level. The 1.953 difference between the equal additions low memory mean and the decomposition low memory mean was not significant

Table 4-4

Means, Standard Deviations, and Differences Betweem Means on the Posttest by Treatment and Memory Classification

Treatment	N	Mean	S.D.	Difference
Low Stress				
High Memory	15	8.200	6.930	2.836
Low Memory	11	5.364	4.738	
Equal Additions				
High Memory	10	6.900	3.755	2.947
Low Memory	13	9.847	6.053	
Decomposition				
High Memory	7	13.858	5.273	2.058
Low Memory	10	11.800	4.516	

(*p*>.05). The difference of 6.437 between the low stress low memory group mean and the decomposition low memory group mean was also significant (*p*<.05).

Aptitude-treatmemt interactions postulated in Hypotheses Three through Five predicted nonsignificant differences in subtraction accuracy on the posttest between the low and high memory groups after receiving instruction in either of the treatments. The analysis of variance for the posttest reported in Table 4-2 indicated a nonsignificant interaction for memory classifications across treatment groups. Table 4-4 presents the posttest means, standard deviations, and the differences between the means for the high and low memory classified groups for each treatment.

Tests of Hypotheses

From this analysis of the data, the following conclusions can be made for this sample:

H_1 Rejected--There was a significant difference between treatment groups on the posttest.

H_2 Rejected--For low memory classified students there was a significant difference between the treatment groups on the posttest performance.

H_3 Accepted--There was no significant difference between low and high short term memory groups on the posttest after instruction on the decomposition subtraction algorithm.

H_4 Accepted--There was no significant difference between low and high short term memory groups on the posttest after instruction on the equal additions subtraction algorithm.

H_5 Accepted--There was no significant difference between low and high short term memory groups on the posttest after instruction on the low stress subtraction algorithm.

Interpretation of the Data

The analysis of variance for subtraction inventory performance, SAT computational score and IQ indicated that each of the treatment groups analyzed were homogeneous for general computational skills, IQ, and subtraction skill as measured by the subtraction inventory at the onset of the treatment intervention. The analysis indicated that the memory classifications were only significantly different on the dependent variable IQ and not for the dependent variables of subtraction accuracy or general computational skills for this sample. The children in this study demonstrating difficulty with subtraction were equally represented in both high and low short term memory classifications.

The analysis of variance on the posttest scores indicated that there were significant differences in subtraction skills as measured by this instrument after receiving a short period of instruction in either low stress subtraction, equal additions subtraction algorithm, and the traditional decomposition algorithm. There was no significant difference between the posttest scores for memory classifications nested within each treatment group. The mean scores on the posttest indicated that the decomposition treatment group had the highest number of problems correct, followed by the equal additions treatment and the low stress treatment groups. Post hoc analysis revealed a significant difference between both the low stress and the equal additions compared to the decomposition posttest means. The difference between the equal additions and the low stress means was not statistically significant at the 95% level of confidence.

The aptitude-treatment interactions were insignificant in their effect upon posttest performance in this study. It appears that instruction in low stress subtraction algorithm or in equal additions algorithm does not have a differential effect upon subtraction skill for the students in this study with high or low short term memory capacity.

The findings do present some interesting indications and results from the treatment interventions and the short term memory aptitude interactions. Students in the decomposition treatment demonstrated 61% mastery of multi-digit subtraction with intermittent borrowing across xeroes. The equal additions group demonstrated 43% mastery and the low stress group demonstrated only 35% mastery of this skill. For this sample, the short three week intervention in the decomposition subtraction algorithm presented in a highly sequential format that produced greater levels of mastery than the equal additions algorithm or the low stress algorithm.

Posttest Error Analysis

None of the treatment groups demonstrated total mastery of multi-digit subtraction with borrowing across zeroes. An analysis of the errors made on the posttest was made to determine what types of errors were made and to determine if any pattern of errors occurs for each of the treatment groups. Table 4-5 presents the types of errors identified on the posttest with the error frequency per student for each treatment.

Table 4-5

Posttest Subtraction Errors Per Student

Error	Low Stress	Equ Add	Decomp
1. Does not replace a zero with a nine when borrowing across zeroes.	4.88	.973	.755
2. Does not decrease a column borrowed from.	3.57	.736	1.155
3. Decrements unnecessarily.	4.64	.026	1.55
4. Replaces a zero with a nine instead of overloading the column $(^{1}0)$.	4.75	.016	.155
5. $0 - X = 0$	0.642	.289	.022
6. $0 - X = X$	0.140	.421	.000
7. Top from bottom.	0.210	.210	.000
8. Basic fact errors.	0.210	.974	1.220
9. Problem not attempted.	0.935	6.368	.244
10. Errors unaccounted for.	0.039	.016	.244

Almost every type of error identified was represented in each of the treatment groups. Some error types were more prevalent than others. Each treatment group had higher numbers of type 1, 2, 3 and 4 errors than any other type.

The equal additions treatment group demonstrated errors in not replacing zeroes with nines when borrowing across zeroes, not decreasing a column borrowed from, $0 - X = 0$ type errors, top from bottom and basic fact errors. This treatment group also had the greatest incidence of not attempting a problem than any of the other treatment groups. There were several students who attempted to utilize the equal additions algorithm with three and four digit problems but then changed to the decomposition algorithm for problems with more than four digits. The equal additions algorithm was not utilized by anyone in the treatment group with problems larger than four digits. There were also more $0 - X = 0$ errors with this treatment group than with any of the other treatment groups.

The error analysis of the posttest performance for the decomposition treatment group identified the errors of not replacing a zero with a nine when borrowing across zeroes, not decreasing a column which is borrowed from, subtracting top from bottom, basic fact errors, problems not attempted, and unaccounted for errors. Errors which could not be accounted for were more prevalent in this treatment group than in any other. Most of the decrement errors were decreasing unnecessarily rather than failing to decrease a number from the borrowed column.

The low stress treatment group demonstrated significant difficulty in discriminating when to decrement and when not to decrement. There was an almost equal number of incidents where students decreased a column when unnecessary as well as incidents where they failed to decrease a column when necessary. Students also demonstrated difficulty in discriminating when to replace a zero with a nine and when it is correct to subtract from a zero $(0 - 0 = 0)$. An error which was observed only once in the other treatment groups but occurred several times in the low stress treatment was perseveration in changing zeroes to nines with strings of zeroes where the last zero should be an overloaded place value $({}^{1}0)$. It is interesting to note that the low stress group had the lowest rate of basic fact errors than the other two treatment groups.

Summary

The analysis of the findings in this study of the effects of a brief treatment utilizing three subtraction algorithms on subtraction skills for students demonstrating the need for some remedial intervention in subtraction and with high and low short term memory was presented in this chapter. The statistical analysis procedures Means and General Linear Models from the SAS computer programs were utilized in this analysis. After a series of fifteen, fifteen to thirty minute lessons in either the decomposition algorithm, equal additions algorithm, or low stress subtraction algorithm, there was a significant difference between treatment groups on a posttest measure of subtraction skill. The decomposition treatment group had a statistically significant higher mean score than the equal additions and the low stress treatment groups. There did not appear to be any significant differences between high and low short term memory students in any of the within treatment groups on posttest performance. There was, however, a significant difference between the low memory groups across treatments with the same pattern prevailing as with both high and low memory groups together.

An analysis of the errors made on the posttest indicated that most of the errors were involving decrements and borrowing across zeroes for all treatment groups. The decomposition group had the greatest number of unaccounted for errors as well as failures to decrease a column which is borrowed from. The equal additions group demonstrated a higher prevalence of failure to attempt larger subtraction problems. The equal additions algorithm was also prone to $0 - X = 0$ types of errors. The low stress algorithm tended to confuse the students as to when a column should be, or should not be decreased and when to change a zero to a nine. This algorithm tended to focus attention to these discriminations. It was noted that while more errors were made utilizing this algorithm on the posttest, these errors were more concentrated than the other treatment groups.

CHAPTER V

CONCLUSIONS, IMPLICATIONS, AND RECOMMENDATIONS

The mastery of the four basic computational algorithms is viewed as essential for meeting the basic competencies required of all school children. Yet, there has been considerable concern that some children are not mastering these basic skills. Recent test scores on national samples have been steadily declining, indicating to many that our curriculums may be "watered down" by more popular areas of study and that the schools' effectiveness to teach the basic skills has been seriously eroded.

Efforts aimed at meeting this area of concern have generally centered upon more accurate measurement of the mastery levels of children at various milestones in school. This minimum competency movement has put more pressure upon teachers and children to meet these demands, but it has failed to explore more effective methods of performing and learning those skills. It is apparent that the most efficient and effective method of developing these skills is needed to attain the high standards of mastery being demanded from our children.

Research on the difficulties children have with computational skills has identified more problems with the subtraction algorithm than with any other (Cox, 1975). There has been a great deal of research conducted on the difficulties children encounter with the subtraction algorithm (Brown & Burton, 1978; Resnick, 1982). Complex theories as to how children generate their irregular procedures have been developed. Surprisingly, though, there has not been any extensive research on alternative subtraction algorithms since Brownell's studies comparing the equal additions method and the decomposition method in the 1940's.

One notable exception to this state of affairs is the work of Hutchings (1976) on a set of algorithms called "low stress" algorithms. These procedures have been developed with the intention of diminishing the demands placed upon short term memory as much as possible in order to facilitate computational execution and learning. Research conducted on these procedures has indicated that low stress algorithms do facilitate computational skill in addition, multiplication and subtraction for disadvantaged in long term treatment programs and for intellectually handicapped students on easy subtraction. Their effectiveness as a short term remedial intervention for difficult subtraction has not been investigated.

The purpose of this study has been to investigate the effectiveness of Hutchings' low stress subtraction algorithm compared to the equal additions and the decomposition algorithms with students with high and low short term memory aptitudes. The Null Hypotheses predicted that there would be no differences between subtraction skills after receiving instruction in either low stress subtraction, equal additions subtraction, or decomposition subtraction procedures for students with high or low short term memory aptitudes.

The treatments consisted of a fifteen lesson intervention with classroom instruction and a worksheet each day by the regular classroom teacher, consuming approximately twenty minutes of classroom time each day. The sample consisted of sixty-six fifth grade students in six classes who demonstrated questionable mastery of multi-digit subtraction on a subtraction inventory. The sample was drawn from two schools were students are randomly assigned to classrooms at the beginning of each school year. One intact class in each school was randomly assigned to each of the three treatments. The teachers were provided all the materials for the lessons in each treatment and individual instruction in each procedure. All grading of papers was done by the experimenter, and feedback was given to the teachers about errors the students were making. Each treatment consisted of fifteen worksheets providing students an opportunity to practice the skill presented by the teacher. Each treatment was task analyzed and presented in a sequential hierarchical order.

Control variables of subtraction inventory scores, Stanford Achievment Test computational subtest scores, and Otis-Lennon IQ scores were employed as a measure of group homogeneity. An analysis of variance indicated that there were no significant differences between treatment groups for these control variables with this sample. Students were identified as having questionable mastery of multi-digit subtraction if their score on the subtraction inventory was less than 80%. Only those students identified as remedial candidates were included in the data analysis. Short term memory was measured on the Visual Aural Digit Span Test (Koppitz, 1977). Students receiving a score at or above the median score for the entire sample were assigned to the high memory classification. Students scoring below the median were assigned to the low memory classification. A multi-digit, twenty-item subtraction posttest was administered to the entire sample immediately following the end of the treatment.

Data analysis consisted of an analysis of variance for nested within groups variables. The analysis model was of a mixed design with random assignment of students to classes, and fixed treatment and memory classification. The level of significance was set at .05 for rejection of the Null Hypothesis. A post hoc analysis of treatment means was performed utilizing Scheffe's Multiple Comparisons procedure to determine significant differences between the posttest means for each treatment. Aptitude-treatment interactions were also analyzed with the Multiple Comparisons procedure to examine differences on the posttest means for the high and low memory classifications for each treatment.

The analysis of variance did find a significant difference between posttest performances for the treatment groups. The simple effects analysis indicated that the low stress and equal additions treatments did significantly ($p<.05$) poorer than the decomposition treatment on the posttest. The analysis did not indicate a significant aptitude-treatment interaction for any of the treatments with short term memory. The post hoc analysis also indicated that the difference between the means of the high and low memory students was not statistically significant for any of the treatments. Differences in posttest means for the low memory classifications followed the same pattern exhibited by the treatment means for both high and low memory classifications combined. The analysis indicated that for the low memory classification alone, both the equal additions and the decomposition performed significantly ($\underline{p}<.05$) better than the low stress treatment.

An analysis of the errors made on the posttest for each treatment indicated a pattern of errors specific to each treatment. The low stress treatment demonstrated more problems discriminating between when and when not to decrease the next column. The equal additions treatment tended to make more $0 - X = 0$ errors than the other treatments. The decomposition treatment, along with demonstrating fewer errors, made more random unsystematic errors which could not be accounted for in terms of the algorithm than the other treatments. The decomposition treatment also demonstrated errors in decreasing columns borrowed from, especically if that column was separated from the operating column by two or more columns with zeroes which had to be borrowed across. Students in the equal additions treatment, without exception, either stopped or reverted to the decomposition algorithm when they encountered problems greater then four digits on the posttest.

Conclusions

From the results of this study, it would appear that instruction in an alternative subtraction algorithm as a short term remedial intervention may not be as effective as a sequential review of the traditional decomposition algorithm. The introduction of an alternative computational method has been suggested as a remedial approach with students who are experiencing difficulty with a computational algorithm (Ashlock, 1976). The results of this study indicate that this practice may produce a deterioration in the computational skill while the student attempts to learn the new procedure.

Previous studies (Kalin and Shami, 1975; Lester, 1978) found the alternative algorithm to be very effective in facilitating subtraction accuracy. These studies were conducted with early elementary students and focused upon learning the basic subtraction facts and the subtraction problems appropriate for third grade students. Bright (1978) has indicated that difficult subtraction with borrowing across xeroes is mastered in the upper elementary grades. Another difference between the previous studies and this one is that the alternative algorithms were taught as the primary subtraction algorithm and not as a supplemental procedure. The students in this study were presented this material as a supplement to their regular arithmetic lessons.

It has been indicated (Hutchings, 1972; Boyle, 1973) that one of the features of the low stress algorithm is its reduced demands upon short term memory. In this study it was indicated that in a short treatment duration there was no advantage in the low stress procedure with students demonstrating below average short term memory aptitude.

Heavy demands were placed upon short term memory for learning the basic procedure as evidenced in the considerable confusion with the borrowing processes. It appears that concentrated effort must be placed upon recalling the rules for making the discriminations dealing with regrouping. There are two conditions involving zeroes which require variations in the procedure. Students are required to discriminate between when to overload a place value and when overloading is not necessary. The students in this study found it difficult to make this discrimination as well as when to replace a zero with a nine or when to leave the zero unchanged. Also, some students consistently decreased the right-most digit rather than just copy it into the space for the renamed minuend when beginning the renamining procedure.

Short term memory has been considered a primary factor in performing arithmetic computations (Case, 1982). Theoretically, if the demands on short term memory and central processing capacity can be reduced, it would be possible to more effectively teach those students with impaired memory capacity. This aptitude-treatment interaction is very difficult to obtain. The division of the VADS scores at the median score resulted in significantly large and equal group sizes in the high and low memory classifications which was desirable for statisical analysis. This also created a comparison which may not have been that accurate. The majority of VADS scores were clustered close to the median which meant that the differences between VADS scores for the high and low classifications in many cases were within four points. Thirty-four students had VADS scores within two points either about or below the median score. It may be necessary to include other aptitudes in the equation before effective aptitude-treatment interactions can be observed. Other factors might include knowledge of the number system, personality traits, cognitive development, and style.

Implications

While the low stress subtraction algorithm does appear to be a more efficient subtraction procedure, it has not demonstrated that in a three week intervention that it can be mastered when taught by regular classroom teachers who have had no previous experience with the procedures. Previous studies (Lester, 1973; Kalin and Shami, 1975) have demonstrated the effectiveness of the low stress procedure over an extended intervention period. The low stress procedure may be more effective in an extended intervention which focuses upon the discriminations required by the procedure. Once the basic procedure is mastered, it may indeed demand less working memory thus facilitating the subtraction accuracy of students with diminished short term memory aptitudes. In a remedial setting with experienced clinicians, the low stress procedure may be an effective short term remedial intervention. Although the teachers in this study exhibited interest and enthusiasm for the low stress procedures, their lack of experience in teaching these procedures may contribute to their effectiveness as a remedial procedure.

The equal additions subtraction algorithm was found to be too cumbersome for subtraction problems with more than three digits. The students did, however, find it interesting and enjoyed using the procedure for two and three digit problems. The equal additions subtraction algorithm does appear to be of merit as an alternative algorithm for subtraction, but it does not appear to be an effective procedure for large multi-digit problems

Recommendations

The results of this study indicate a need for more prolonged investigations with alternative subtraction algorithms at various levels of skill dificulty. None of the treatment interventions fully eliminated errors in subtraction. The optimum intervention duration is an area which appears to need further investigation. It may be that a little longer intervention with the low stress treatment could increase students' performances beyond the levels achieved by the decomposition intervention with the same duration. The treatment content might be a factor which could improve the low stress algorithm. More emphasis upon the discriminations demanded by the procedures might improve the effectiveness of the algorithm as a short term remedial procedure. With this knowledge about the problems students had in learning the low stress subtraction algorithm, future interventions can be designed to focus more upon these difficult discriminations making the intervention more efficient.

The aptitude-treatment aspect in remedial procedures could be investigated further using other factors. Short term memory may be a factor in poor arithmetic achievement, but may not be a significant factor in the effectiveness of a remedial procedure. There may be other factors which may interact more with certain methods of computation, such as knowledge of the number system, cognitive style, or cognitive development. Future investgations might explore the interactions of these aptitudes with the treatment interventions to discover if certain methods are more effective with students exhibiting certain characteristics.

The equal additions algorithm in this study appears to have had little effect upon the computational skill of multi-digit subtraction except that many students were frustrated in their attempts to apply it to problems involving more than four digits. In this respect, this treatment had little or no utility on posttest performances. The students who did revert to the decomposition algorithm did so without the benefit of any treatment interventions which could be utilized in that procedure. Future studies may be able to utilize this non-utilitaian effect as a control treatment in comparison with other treatments to avoid differential effects from a control group without any interventions.

APPENDIX A

ANALYSIS OF VARIANCE FOR SUBTRACTION INVENTORY

Source	Df	ANOVA SS	MS	F	P
Treatment	2	46.963	23.482	0.500	<.20.
Treatment/class	3	140.854	46.951	2.693	.06
Memory	1	5.115	5.115	0.519	<.20.
Trt*Memory	2	12.069	6.034	0.613	<.20.
Class/Mem (Trt)	3	29.541	9.847	0.565	<.20.
Error	58	1010.828	17.428		

APPENDIX B

ANALYSIS OF VARIANCE FOR SAT COMPUTATIONAL GRADE LEVELS

Source	Df	ANOVA SS	MS	F	P
Treatment	2	3.290	1.645	0.820	<.20
Treatment/Class	3	6.014	2.004	1.207	<.20
Memory	1	0.502	0.502	0.302	<.20
Trt*Memory	2	0.458	0.229	0.138	<.20
Class/Mem (Trt)	3	4.980	1.660	1.406	<.20
Error	57	67.355	1.181		

APPENDIX C

ANALYSIS OF VARIANCE FOR IQ

Source	Df	ANOVA SS	MS	F	P
Treatment	2	41.600	20.800	0.594	<.20
Treatment/Class	3	104.988	34.996	0.252	<.20
Memory	1	1410.292	1410.292	20.204	.025
Trt*Memory	2	300.139	150.069	2.147	<.20
Class/Mem (Trt)	3	209.599	69.866	1.984	<.20
Error	57	7900.857	138.612		

APPENDIX D

SUBTRACTION INVENTORY

SUBTRACTION INVENTORY

Name______________________

910533 -250529	487792 -456846	746825 -546359	752326 -386306
679513 -194042	991640 -939281	427600 -376666	715884 -113830
313458 -213642	403237 -174594	599067 -568776	240001 -176614
988055 -728097	674511 -139096	518840 -485849	800817 -322882
991491 -733899	223558 -175847	192817 -168921	469243 -453926

APPENDIX E

POSTTEST

POSTTEST

Name__________________

(1) 589
-132

(2) 462
-183

(3) 627041
-132051

(4) 5000
-1476

(5) 1406201
-1399202

(6) 9200304160
-1400403241

(7) 47007012301
-21006121421

(8) 62734761021
-43641850012

(9) 90120913004
-21630821302

(10) 41007210031
-12004324021

(11) 7120300421
-6130120511

(12) 800012301004
-210113241003

POSTTEST

Page 2

(13) 400000
-210201

(14) 692012003
-451001201

(15) 36200001
-18702001

(16) 721000001
-436002001

(17) 781300040001
-692401030200

(18) 478263041
-234687094

(19) 260001020001230
-170406010302120

(20) 49216001209286000030006
-29345003208679001020605

APPENDIX F

DECOMPOSITION WORKSHEETS

Decomposition #1 Name___________________

$\begin{array}{r}84\\-\underline{54}\end{array}$	$\begin{array}{r}54\\-\underline{13}\end{array}$	$\begin{array}{r}38\\-\underline{15}\end{array}$	$\begin{array}{r}71\\-\underline{50}\end{array}$	$\begin{array}{r}89\\-\underline{58}\end{array}$
$\begin{array}{r}59\\-\underline{45}\end{array}$	$\begin{array}{r}19\\-\underline{15}\end{array}$	$\begin{array}{r}97\\-\underline{33}\end{array}$	$\begin{array}{r}63\\-\underline{32}\end{array}$	$\begin{array}{r}62\\-\underline{41}\end{array}$
$\begin{array}{r}50\\-\underline{39}\end{array}$	$\begin{array}{r}29\\-\underline{18}\end{array}$	$\begin{array}{r}51\\-\underline{31}\end{array}$	$\begin{array}{r}98\\-\underline{52}\end{array}$	$\begin{array}{r}45\\-\underline{32}\end{array}$
$\begin{array}{r}54\\-\underline{23}\end{array}$	$\begin{array}{r}67\\-\underline{27}\end{array}$	$\begin{array}{r}29\\-\underline{14}\end{array}$	$\begin{array}{r}86\\-\underline{44}\end{array}$	$\begin{array}{r}93\\-\underline{53}\end{array}$
$\begin{array}{r}97\\-\underline{73}\end{array}$	$\begin{array}{r}26\\-\underline{12}\end{array}$	$\begin{array}{r}28\\-\underline{25}\end{array}$	$\begin{array}{r}96\\-\underline{20}\end{array}$	$\begin{array}{r}89\\-\underline{51}\end{array}$

Decomposition #2

Name__________________

39 - 25 =

60 - 40 =

47 - 22 =

64 - 32 =

69 - 51 =

75 - 35 =

61 - 30 =

85 - 33 =

54 - 53 =

98 - 23 =

53 - 33 =

24 - 15 =

86 - 24 =

98 - 26 =

Decomposition #3　　Name________________

27356 - 134	14995 - 841	83384 - 253
41738 - 715	65578 - 435	79843 - 643
29868 - 247	87690 - 540	80208 - 103
94791 - 291	43977 - 834	70697 - 671
19629 - 517	42970 - 430	86203 - 101
48183 - 163	10430 - 430	90912 - 612

Decomposition #4

Name____________________

91 - 6	43 - 7	71 - 3	84 - 5	69 - 9	40 - 4
97 - 8	33 - 6	54 - 5	64 - 6	91 - 2	20 - 8
62 - 6	62 - 3	60 - 7	14 - 5	39 - 7	65 - 7
64 - 8	22 - 4	21 - 5	45 - 8	22 - 2	83 - 6
51 - 9	48 - 8	52 - 7	45 - 6	17 - 9	63 - 4
38 - 9	86 - 9	51 - 8	51 - 3	33 - 4	34 - 8

Decomposition # 5

Name____________________

19 - 8 =

45 - 7 =

51 - 3 =

73 - 6 =

23 - 9 =

54 - 5 =

55 - 6 =

37 - 9 =

80 - 4 =

25 - 9 =

77 - 9 =

91 - 6 =

65 - 6 =

74 - 8 =

Decomposition # 6

Name___________________

36 - 7 =

43 - 8 =

91 - 5 =

64 - 6 =

73 - 6 =

34 - 0 =

40 - 7 =

32 - 4 =

30 - 4 =

87 - 8 =

26 - 8 =

86 - 9 =

55 - 7 =

34 - 8 =

Decomposition # 7

Name_________________

31	47	33	88	46	15
- 3	- 8	- 4	- 9	- 9	- 7
24	46	35	82	68	46
- 5	- 8	- 9	- 5	- 9	- 7
62	96	34	17	70	21
- 8	- 9	- 5	- 8	- 3	- 2
21	40	56	79	25	92
- 4	- 7	- 9	- 0	- 8	- 4
39	22	34	55	68	27
- 9	- 3	- 9	- 6	- 9	- 9
76	34	24	84	81	21
- 7	- 8	- 5	- 9	- 3	- 5

Decomposition # 8

Name________________

21 -26	53 -28	73 -15	55 -18	87 -69
87 -28	63 -34	76 -26	93 -38	42 -17
92 -23	54 -44	79 -64	46 -24	87 -39
53 -47	81 -62	40 -24	61 -36	92 -79
86 -28	65 -48	44 -15	73 -37	92 -75

Decomposition # 9

Name__________________

88 - 79 =

65 - 46 =

64 - 37 =

85 - 37 =

79 - 0 =

53 - 28 =

22 - 19 =

40 - 24 =

28 - 19 =

53 - 33 =

62 - 46 =

47 - 29 =

55 - 48 =

72 - 23 =

Decomposition # 10

Name____________________

301	208	504	307
− 26	− 29	− 56	− 88
200	509	804	604
− 31	− 76	− 68	− 36
603	206	901	704
− 46	− 69	− 32	− 78
905	503	202	704
− 46	− 69	− 56	− 36
906	700	307	704
− 37	− 62	− 69	− 36

Decomposition # 11

Name____________________

207 - 29 =

304 - 87 =

703 - 66 =

905 - 46 =

801 - 69 =

202 - 14 =

508 - 76 =

304 - 36 =

600 - 76 =

705 - 77 =

400 - 74 =

601 - 45 =

300 - 69 =

807 - 39 =

Decomposition # 12 Name____________________

708	402	508	409
-349	-476	-219	-169
304	806	502	705
-185	-219	-113	-145
509	600	602	700
-152	-273	-147	-549
903	701	305	804
-344	-105	-247	-449
601	308	701	308
-235	- 69	-339	-139

Decomposition # 13

Name____________________

707	407	606	701
-239	-338	-578	-366
408	401	609	302
- 39	-288	-437	-186
604	403	700	207
-135	-267	-179	- 99
702	303	501	700
-135	-278	-299	-548
209	700	601	503
-143	-567	-393	-429

Decomposition # 14 Name____________________

3197	6303	4772	6346
-2658	-4804	-2825	-3527
6593	7732	6213	8151
-5818	-5927	-3307	-3432
6144	2341	8167	9185
-5839	-1453	-2448	- 2269
8542	5744	3587	7116
-5616	-1839	-1959	-5307
5793	1894	9273	9280
-4856	-1657	-3637	-5524

Decomposition # 15   Name ____________________

$$\begin{array}{rrrr} 2001 & 4600 & 9010 & 6006 \\ -\underline{1243} & -\underline{3165} & -\underline{7364} & -\underline{4739} \end{array}$$

$$\begin{array}{rrr} 87004 & 207060 & 8900607 \\ -\underline{17889} & -\underline{136047} & -\underline{4600506} \end{array}$$

$$\begin{array}{rrr} 520001 & 714000 & 4140034 \\ -\underline{49882} & -\underline{245000} & -\underline{2440044} \end{array}$$

$$\begin{array}{rrr} 900851 & 960301 & 60000 \\ -\underline{267596} & -\underline{385143} & -\underline{49671} \end{array}$$

APPENDIX G

EQUAL ADDITIONS WORKSHEETS

Equal Additions # 1 Name________________

1. When you have a subtraction problem that requires regrouping, try adding a number to both the top and bottom numbers which will make it easier.

For example,

40 = (40 + 1) = 41
-29 = (29 + 1) =-30
11

Try these:

(a) 14 What number would (14 + __<---Keep in
-5 make this easier? ------> (5 + __ <---mind that
these must be the
SAME!!!

(b) 12 (12 + __)<--Are they (c) 11 (11 + __)->___
-3 (3 + __)<--the SAME? -6 (6 + __)->__

(d) 16 (16 + __)-->____ (e) 21 (21 + __)-->___
-9 (9 + __)-->____ -16 (16 + __)-->___

Equal Additions #2 Name____________________

Review: When you have a subtraction problem that requires borrowing, ADD instead! Add a number to both the top and bottom that will make it an easier problem. Remember the same nummber is added to both top and bottom.

Try these.

(1) 71 (71 +__)=____
- 3 (3 +__)=____

(2) 84 (84 +___)=____
- 5 (5 +___)=____

What number here ^ will make 3+__= 10?

(3) 40 (40 +__)=____
- 4 (4 +__)=____

(4) 54 (54 +___)=____
- 5 (5 +___)=____

(5) 91 (91 +__)=____
- 2 (2 +__)=____

(6) 62 (62 +___)=____
- 3 (3 +___)=____

(7) 60 (60 +__)=____
- 1 (1 +__)=____

(8) 70 (70 +___)=____
- 2 (2 +___)=____

Equal Additions #3

Name ____________________

(1) 91 (91 +__)=_____
- 6 (6 +__)=_____

(2) 43 (43 +___)=_____
-7 (7 +___)=_____

(3) 97 (97 +__)=_____
- 8 (8 +__)=_____

(4) 33 (33 +___)=_____
-6 (6 +___)=_____

(5) 64 (64 +__)=_____
- 6 (6 +__)=_____

(6) 20 (20 +___)=_____
-9 (9 +___)=_____

(7) 62 (62 +__)=_____
- 6 (6 +__)=_____

(8) 60 (60 +___)=_____
-7 (9 +___)=_____

(9) 85 (85 +__)=_____
- 7 (7 +__)=_____

(10) 64 (64 +___)=_____
-8 (8 +___)=_____

(11) 45 (45 +__)=_____
- 8 (8 +__)=_____

(12) 22 (22 +___)=_____
-9 (5 +___)=_____

Equal Additions #4

Name ________________

(1) 83 (83 +__)=____
- 6 (6 +__)=____

(2) 51 (51 +___)=____
- 9 (9 +___)=____

(3) 52 (52 +__)=____
- 7 (3 +__)=____

(4) 17 (+)=____
- 9 (+)=____

(5) 38 (+)=____
- 9 (+)=____

(6) 86 (+)=____
- 9 (+)=____

(7) 34 (+)=____
- 9 (+)=____

(8) 54 (+)=____
- 6 (+)=____

(9) 36 (+)=____
- 8 (+)=____

(10) 41 (+)=____
- 3 (+)=____

(11) 75 (+)=____
- 6 (+)=____

(12) 37 (+)=____
- 9 (+)=____

Equal Additions #5 Name________________

(1) 74 (____+____)=________
- 8 (____+____)=________

(2) 83 ____+____=________
- 9 ____+____=______

(3) 85 (____+____)=________
- 6 (____+____)=________

(4) 54 ____+____=________
- 8 ____+____=______

(5) 71 (____+____)=________
- 3 (____+____)=________

(6) 77 ____+____=________
- 8 ____+____=______

(7) 63 (____+____)=________
- 4 (____+____)=________

(8) 78 ____+____=________
- 9 ____+____=______

(9) 66 (____+____)=________
- 9 (____+____)=________

(10) 35 ____+____=________
- 7 ____+____=______

(11) 84 (____+____)=________
- 5 (____+____)=________

(12) 86 ____+____=________
- 8 ____+____=______

Equal Additions #6

Name__________________________

(1) $\begin{array}{r} 15 \\ -\underline{9} \end{array}$

(2) $\begin{array}{r} 52 \\ -\underline{5} \end{array}$

(3) $\begin{array}{r} 28 \\ -\underline{9} \end{array}$

(4) $\begin{array}{r} 66 \\ -\underline{7} \end{array}$

(5) $\begin{array}{r} 42 \\ -\underline{8} \end{array}$

(6) $\begin{array}{r} 86 \\ -\underline{9} \end{array}$

(7) $\begin{array}{r} 14 \\ -\underline{5} \end{array}$

(8) $\begin{array}{r} 47 \\ -\underline{8} \end{array}$

(9) $\begin{array}{r} 10 \\ -\underline{3} \end{array}$

(10) $\begin{array}{r} 81 \\ -\underline{2} \end{array}$

(11) $\begin{array}{r} 51 \\ -\underline{4} \end{array}$

(12) $\begin{array}{r} 70 \\ -\underline{7} \end{array}$

Equal Additions #7 Name______________________

(1) 41 (+4)= 45 -36 (+4)= -40 5	(2) 43 (+2)= 45 -18 (+2)= -20 25	(3) 43 (+5)= 48 -35 (+5)= 40 8
(4) 25 (+)= 25 -18 (+)= -20	(5) 97 (+)= -59 (+)= -__	(6) 37 (+)= -18 (+)=
(7) 83 (+)= -74 (+)= -__	(8) 86 (+)= -16 (+)= -__	(9) 83 (+)= -48 (+)=
(10) 32 (+)= -27 (+)= -__	(11) 62 (+)= -53 (+)= -__	(12) 91 (+)= -54 (+)=
(13 89 (+)= -74 (+)= -__	(14) 96 (+)= -57 (+)= -20	(15) 37 (+)= -19 (+)=

Equal Additions # 8

Name ____________________

$\begin{array}{r} 63 \\ -\underline{27} \end{array}$ $\begin{array}{r} 81 \\ -\underline{22} \end{array}$ $\begin{array}{r} 50 \\ -\underline{24} \end{array}$

$\begin{array}{r} 51 \\ -\underline{26} \end{array}$ $\begin{array}{r} 52 \\ -\underline{29} \end{array}$ $\begin{array}{r} 76 \\ -\underline{43} \end{array}$

$\begin{array}{r} 75 \\ -\underline{38} \end{array}$ $\begin{array}{r} 34 \\ -\underline{15} \end{array}$ $\begin{array}{r} 93 \\ -\underline{17} \end{array}$

$\begin{array}{r} 72 \\ -\underline{25} \end{array}$ $\begin{array}{r} 34 \\ -\underline{69} \end{array}$ $\begin{array}{r} 93 \\ -\underline{24} \end{array}$

$\begin{array}{r} 95 \\ -\underline{86} \end{array}$ $\begin{array}{r} 68 \\ -\underline{29} \end{array}$ $\begin{array}{r} 54 \\ -\underline{17} \end{array}$

Equal Additions # 9 Name ______________________

Subtraction with three digit numbers.

STEP 1 STEP 2 SUBTRACT

903 + ___ = ______ + 0 = _________
− 36 + ___ = ______ + 0 = _________

306 + ___ = ______ + 0 = _________
− 39 + ___ = ______ + 0 = _________

501 + ___ = ______ + 0 = _________
− 12 + ___ = ______ + 0 = _________

604 + ___ = ______ + 0 = _________
− 48 + ___ = ______ + 0 = _________

705 + ___ = ______ + 0 = _________
− 36 + ___ = ______ + 0 = _________

203 + ___ = ______ + 0 = _________
− 99 + ___ = ______ + 0 = _________

502 + ___ = ______ + 0 = _________
− 65 + ___ = ______ + 0 = _________

Equal Additions # 11 Name ____________________

STEP 1 STEP 2 Step 3 Subtract

608 + ___ = _____ + ____0 = ______ + ____00 = ______

-449 + ___ = _____ + ____0 = ______ + ____00 = ______

502 + ___ = _____ + ____0 = ______ + ____00 = ______

-175 + ___ = _____ + ____0 = ______ + ____00 = ______

409 + ___ = _____ + ____0 = ______ + ____00 = ______

-169 + ___ = _____ + ____0 = ______ + ____00 = ______

204 + ___ = _____ + ____0 = ______ + ____00 = ______

-185 + ___ = _____ + ____0 = ______ + ____00 = ______

906 + ___ = _____ + ____0 = ______ + ____00 = ______

-319 + ___ = _____ + ____0 = ______ + ____00 = ______

906 + ___ = _____ + ____0 = ______ + ____00 = ______

-503 + ___ = _____ + ____0 = ______ + ____00 = ______

905 + ___ = _____ + ____0 = ______ + ____00 = ______

-559 + ___ = _____ + ____0 = ______ + ____00 = ______

209 + ___ = _____ + ____0 = ______ + ____00 = ______

-162 + ___ = _____ + ____0 = ______ + ____00 = ______

Equal Additions # 11 Page 2

800 + ___ = _____ + ___0 = ______ + ___00 = _______

-273 + ___ = _____ + ___0 = ______ + ___00 = _______

802 + ___ = _____ + ___0 = ______ + ___00 = _______

-113 + ___ = _____ + ___0 = ______ + ___00 = _______

300 + ___ = _____ + ___0 = ______ + ___00 = _______

-145 + ___ = _____ + ___0 = ______ + ___00 = _______

703 + ___ = _____ + ___0 = ______ + ___00 = _______

-344 + ___ = _____ + ___0 = ______ + ___00 = _______

601 + ___ = _____ + ___0 = ______ + ___00 = _______

-104 + ___ = _____ + ___0 = ______ + ___00 = _______

305 + ___ = _____ + ___0 = ______ + ___00 = _______

-147 + ___ = _____ + ___0 = ______ + ___00 = _______

804 + ___ = _____ + ___0 = ______ + ___00 = _______

-649 + ___ = _____ + ___0 = ______ + ___00 = _______

903 + ___ = _____ + ___0 = ______ + ___00 = _______

- 36 + ___ = _____ + ___0 = ______ + ___00 = _______

903 + ___ = _____ + ___0 = ______ + ___00 = _______

- 36 + ___ = _____ + ___0 = ______ + ___00 = _______

Equal Additions # 12

Name ______________________

(1) 401

$-\underline{225}$

(2) 108

$-\underline{\ 49}$

(3) 601

$-\underline{234}$

(4) 308

$-\underline{139}$

(5) 907

$-\underline{139}$

(6) 338

$-\underline{209}$

(7) 506

$-\underline{378}$

Equal Additions #12 Page 2

(8) 701

-226

(9) 108

- 79

(10) 501

-188

(11) 202

-186

(12) 204

-135

(13) 603

-458

(14) 900

-713

(15) 306

-137

Equal Additions # 13 Name ____________________

(1) 986

-669

(2) 356

-174

(3) 406

-169

(4) 656

-168

(5) 249

-191

(6) 761

-275

(7) 821

-347

(8) 695

-567

Equal Additions # 13 Page 2

$$\begin{array}{r} (9)\ 721 \\ -\underline{398} \end{array}$$

$$\begin{array}{r} (10)\ 730 \\ -\underline{290} \end{array}$$

$$\begin{array}{r} (11)\ 935 \\ -\underline{457} \end{array}$$

$$\begin{array}{r} (12)\ 975 \\ -\underline{794} \end{array}$$

$$\begin{array}{r} (13)\ 417 \\ -\underline{378} \end{array}$$

$$\begin{array}{r} (14)\ 508 \\ -\underline{334} \end{array}$$

$$\begin{array}{r} (15)\ 350 \\ -\underline{294} \end{array}$$

Equal Additions # 14 Name ____________________

(1) 3020

-2658

(2) 6000

-4804

(3) 4001

-3006

(4) 7060

-4041

(5) 6270

-4360

(6) 7060

-2140

(7) 5620

-4730

Equal Additions # 14 Page 2

(8) 8151
-2006

(9) 6009
-4208

(10) 12300
- 4789

(11) 862004
-210042

(12) 47006001
-21007000

(13) 90020030
-21030420

(14) 80602030
-50403040

Equal Additions # 15 Name ____________________

(1) 70002

-53071

(2) 90000

-48561

(3) 700201

-160100

(4) 900200

-721301

(5) 260000

-140012

(6) 760001

-130012

(7) 1000000

- 261382

Equal Additions # 15 Page 2

(8) 2160012
-1420012

(9) 4706276
-1200012

(10) 9066000
-7057012

(11) 1000000
- 921002

(12) 90000000
- 1234567

(13) 540000000
-210062012

(14) 600000000
-210003002

APPENDIX H

LOW STRESS SUBTRACTION ALGORITHM WORKSHEETS

Low Stress Worksheet #1 Name ____________________

Overloading a Place Value

When we write a number each column stands for ones, tens, hundreds, etc.. Sometimes when subtracting we have to borrow and then there is more units in a column than normal. One way of representing an "overloaded" column is with a little mark called a "superscript". A superscript looks like this ^{1} . If we borrowed ten units from the next column it might look like this: $^{1}0$, $^{1}7$, $^{1}4$, or $^{1}9$. Practice overloading the ones column in these numbers.

17 = $^{1}7$

13 =

19 =

15 =

18 =

11 =

10 =

What happens to the tens column when you borrow? Don't forget to decrease it by one.

24 = $1^{1}4$

45 =

76 =

93 =

32 =

66 =

14 =

Low Stress Worksheet # 2 Name ___________________

In Low Stress subtraction we rename the minuend before we subtract. The renamed minuend is written under the original so that it is directly above the subtrahend ready to be subtractedall at one time. To begin bring the ones digit down and write it under the original.

$$\begin{array}{cccc} & 5 & 6 & 2 \\ & & & 2 \\ - & 2 & 5 & 8 \end{array}$$

Next look at the number subtracting in that column. Is it larger? If it is, overload the minuend.

$$\begin{array}{cccc} & 5 & 6 & 2 \\ & & & {}^{1}2 \\ - & 2 & 5 & 8 \end{array}$$

Next decrease the next digit in the minuend by one because you borrowed from it. Write the decreased minuend under the original

$$\begin{array}{cccc} & 5 & 6 & 2 \\ & & 5 & {}^{1}2 \\ - & 2 & 5 & 8 \end{array}$$

Again look at the subtrahend under the new renamed number. Is it larger? If it isn't then bring the next digit in the minuend downwithout decreasing it.

$$\begin{array}{cccc} & 5 & 6 & 2 \\ & 5 & 5 & {}^{1}2 \\ - & 2 & 5 & 8 \end{array}$$

Low Stress Worksheet # 2 Page 2

Rename the minuend in these problems. Don't subtract.

6 4 2	7 9 2	9 1 4	4 4 2
- 3 2 6	- 1 5 7	- 5 3 5	- 2 4 5
XXXXXXX	XXXXX	XXXXXX	XXXXXXX
8 5 2	9 9 3	3 1 1	7 4 3
- 1 4 6	- 5 8 4	- 1 5 7	- 5 5 5
XXXXXX	XXXXXXX	XXXXXXX	XXXXXXX
6 4 2	7 1 4	5 8 8	8 3 1
- 2 5 1	- 5 0 6	- 2 5 7	- 1 5 0
XXXXXXX	XXXXXXX	XXXXXXX	XXXXXXX

Low Stress Worksheet # 3 Name ________________

Rename only!

9 5 3 7 5 2 9 5 6 3 1 2 4 7 9 3 1 4 5 2 8 6 3 2 5

- 5 4 6 5 9 6 4 5 5 7 1 1 2 9 8 6 1 2 6 8 9 5 6 1 8

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

7 4 6 7 3 8 5 2 3 5 7 6 4 8 2 9 4 3 6 3 7 9 4 3 2

- 3 5 6 7 3 8 5 5 2 2 9 4 7 8 1 9 6 2 5 8 2 5 7 4 9

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

5 2 7 6 3 8 9 5 4 2 7 6 9 5 4 5 5 4 3 7 9 7 4 6 5

- 2 8 9 5 6 4 6 9 2 7 8 5 9 5 7 3 2 6 4 5 3 6 7 4 2

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

Low Stress # 4 Name ______________________

Rename and then subtract. Remember to do all the Renaming before you subtract!

$$\begin{array}{r} 6\ 5\ 3\ 2\ 2\ 1\ 4\ 7 \\ -\ \underline{4\ 6\ 2\ 9\ 3\ 7\ 2\ 1} \end{array}$$

$$\begin{array}{r} 9\ 8\ 7\ 9\ 6\ 4\ 3\ 2 \\ -\ \underline{5\ 8\ 8\ 9\ 7\ 4\ 3\ 1} \end{array}$$

$$\begin{array}{r} 4\ 7\ 3\ 4\ 9\ 1\ 6\ 8 \\ -\ \underline{2\ 1\ 6\ 5\ 2\ 4\ 5\ 3} \end{array}$$

$$\begin{array}{r} 7\ 1\ 8\ 9\ 6\ 5\ 7\ 2 \\ -\ \underline{6\ 9\ 1\ 3\ 6\ 8\ 9\ 1} \end{array}$$

Low Stress # 5 Name ____________________

Rename and subtract.

$$\begin{array}{r} 8\ 6\ 5\ 4\ 4\ 3\ 7\ 6\ 8\ 4\ 2\ 3\ 4\ 6\ 1\ 5\ 7\ 8\ 8\ 6\ 3\ 4 \\ -\ \underline{3\ 6\ 7\ 6\ 5\ 4\ 8\ 9\ 7\ 5\ 8\ 6\ 3\ 2\ 1\ 4\ 7\ 9\ 9\ 8\ 7\ 5} \end{array}$$

$$\begin{array}{r} 7\ 6\ 4\ 3\ 2\ 4\ 5\ 3\ 5\ 6\ 7\ 4\ 8\ 3\ 2\ 1\ 5\ 6\ 7\ 4\ 8\ 5 \\ -\ \underline{4\ 6\ 4\ 3\ 6\ 7\ 5\ 8\ 6\ 3\ 9\ 6\ 2\ 9\ 8\ 6\ 5\ 8\ 3\ 6\ 8\ 5} \end{array}$$

$$\begin{array}{r} 9\ 8\ 7\ 4\ 3\ 5\ 2\ 7\ 5\ 2\ 1\ 4\ 6\ 7\ 5\ 4\ 8\ 7\ 5\ 9\ 3\ 6 \\ -\ \underline{6\ 9\ 8\ 6\ 3\ 6\ 5\ 2\ 7\ 4\ 1\ 2\ 9\ 8\ 6\ 4\ 8\ 3\ 2\ 7\ 6\ 9} \end{array}$$

Low Stress # 6 Name ____________________

Regrouping with zeroes.

When borrowing across zeroes, simply skip over the zeroes and rename all the other numbers first. Then fill in the empty spaces with nines.

$$\begin{array}{r} 9\;8\;7\;0\;6\;5\;0\;1\;0\;0\;0\;3 \\ 8\,^{1}8\;6\;\;\;^{1}5\,^{1}4\;\;\;^{1}0\;\;\;\;\;\;\;^{1}3 \\ -\;\underline{5\;9\;6\;4\;7\;6\;4\;2\;4\;5\;7\;5} \end{array}$$

$$\begin{array}{r} 9\;8\;7\;0\;6\;5\;0\;1\;0\;0\;0\;3 \\ 8\,^{1}8\;6\;9\,^{1}5\,^{1}4\;9\,^{1}0\;9\;9\;9\,^{1}3 \\ -\;\underline{5\;9\;6\;4\;7\;6\;4\;2\;4\;5\;7\;5} \end{array}$$

Rename and fill in the zeroes with nines. Don't subtract.

$$\begin{array}{r} 5\;4\;0\;8\;0\;0\;9\;1\;6\;0\;0\;6\;5\;4\;0\;0\;6 \\ \\ -\;\underline{2\;1\;8\;7\;2\;6\;5\;3\;2\;1\;6\;5\;8\;7\;6\;7\;8} \end{array}$$

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

$$\begin{array}{r} 9\;7\;3\;0\;0\;8\;6\;0\;0\;0\;0\;0\;0\;0\;0\;0\;2 \\ \\ -\;\underline{5\;6\;4\;8\;6\;7\;5\;4\;7\;9\;4\;2\;5\;7\;8\;6\;4} \end{array}$$

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

Low Stress # 7

Name ____________________

Rename only. Don't forget to skip over zeroes!

9 5 3 0 6 0 0 1

- 6 8 4 6 8 6 4 6

XXXXXXXXXXXXXXXX

5 8 0 7 0 4 0 3

- 2 9 4 8 2 7 8 9

XXXXXXXXXXXXXXXX

6 0 0 0 0 0 0 0

- 2 1 2 3 4 5 6 7

XXXXXXXXXXXXXXXX

8 3 0 0 4 2 0 0 2

- 5 5 5 5 5 5 5 5 5

XXXXXXXXXXXXXXXXX

8 5 4 0 0 7 0 0 9 4

- 6 5 6 4 7 8 6 6 9 5

XXXXXXXXXXXXXXXXXXXX

7 5 0 6 0 4 0 3 0 2

- 3 7 4 9 5 6 2 6 5 7

XXXXXXXXXXXXXXXXXXXX

9 0 0 3 0 0 4 0 0 1

- 4 5 3 6 7 8 9 6 8 6

XXXXXXXXXXXXXXXXXXXX

3 0 0 0 5 2 0 0 0 3 1

- 1 6 7 6 5 6 7 8 5 3 5

XXXXXXXXXXXXXXXXXXXXX

Low Stress # 8 Name ______________________

Rename and Subtract. Skip over the zeroes and replace with nines.

$$\begin{array}{rl} & 8\;1\;6\;9\;0\;8\;2\;7\;6\;3\;0\;5\;7\;4\;0\;2\;1\;0\;0\;0\;3 \\ - & \underline{5\;3\;4\;7\;9\;8\;5\;3\;9\;3\;4\;7\;6\;2\;1\;3\;4\;8\;6\;5\;6} \end{array}$$

$$\begin{array}{rl} & 6\;8\;0\;4\;7\;3\;1\;0\;9\;8\;1\;6\;2\;0\;4\;0\;0\;5\;7\;0\;1 \\ - & \underline{4\;3\;1\;7\;6\;9\;8\;5\;4\;6\;7\;4\;2\;6\;7\;8\;5\;8\;9\;7\;6} \end{array}$$

$$\begin{array}{rl} & 7\;2\;4\;0\;6\;6\;0\;5\;4\;6\;0\;7\;8\;0\;4\;0\;1\;7\;0\;0\;3 \\ - & \underline{2\;6\;8\;7\;4\;3\;6\;5\;2\;4\;8\;7\;6\;9\;5\;1\;3\;4\;5\;6\;7} \end{array}$$

$$\begin{array}{rl} & 4\;2\;0\;7\;6\;0\;8\;9\;0\;4\;3\;0\;1\;6\;5\;0\;0\;0\;0\;0\;4 \\ - & \underline{1\;9\;8\;6\;5\;1\;2\;6\;8\;7\;2\;3\;1\;1\;4\;5\;7\;4\;7\;5\;8} \end{array}$$

Low Stress # 9 Name ____________________

Rename and subtract.

8 9 0 0 9 7 5 0 0 0 0 7 5 4 0 0 6 4 3 0 0 0 6 0 5 0 0 2

- 5 7 5 5 9 9 6 4 4 3 3 9 9 9 8 8 7 7 6 6 4 4 8 9 9 9 6 5

7 8 0 0 0 0 7 8 0 0 5 0 0 0 2 0 2 0 2 0 3 0 4 0 4 0 4

- 3 8 6 4 5 7 9 0 8 7 6 8 9 7 7 9 5 4 6 7 2 3 5 8 9 5 5

8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 7 0 0 0 0 8 0 0 0 0 0 2

- 4 3 4 2 4 6 4 7 6 3 5 7 2 5 7 9 5 6 4 6 9 9 4 1 2 3 6

Low Stress # 10 Name ____________________

Rename and subtract.

$$\begin{array}{r} 652039007051 \\ -\underline{227849458941} \end{array}$$

$$\begin{array}{r} 740003800502 \\ -\underline{487611624320} \end{array}$$

$$\begin{array}{r} 560021007206 \\ -\underline{312510284618} \end{array}$$

$$\begin{array}{r} 5100002460059 \\ -\underline{4021951372268} \end{array}$$

$$\begin{array}{r} 79100043020045 \\ -\underline{52148913826409} \end{array}$$

Low Stress # 11 Name ____________________

Varied Regrouping with zeroes.

There are two kinds of zeroes. Some you borrow across and others you do not borrow across. The kind of zero that you borrow across is replaced with a nine just like the lessons before this one.

For example: 6 0 0 0 2

5 9 9 9^{1}2

- 2 3 3 3 3

The other kind of zero you will find when you borrow to a zero orsubtract zero from zero. For example:

3 0 0 0 4 5 3 5 2 0

2^{1}0 0 0 4 5 3^{1}5 2 0

- 1 2 0 0 4 - 2 3 6 1 0

The key question is: "Can I take the bottom number from the top?"

Rename Only.

7 2 3 0 0 0 7 6 0 0 8 2 0 0 0 4 2 3 0 0 0 0 7 0 2 6 7

- 4 9 1 2 0 0 4 8 2 9 9 1 0 2 0 2 9 9 6 4 8 0 9 0 3 4 8

xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

Low Stress # 12 Name ____________________

Review: Varieds Renaming

Remember- There are two kinds of zeroes. The kind you borrow across and replace with a nine and the kind that you just bring down because they are OK as is. For Example:

$$\begin{array}{r} 2\ 3\ 0\ 0\ 0\ 0\ 0\ 6 \\ 2\ 2\ 9\ 9\ 9\ {}^{1}0\ 0\ 6 \\ -\ \underline{1\ 0\ 4\ 0\ 0\ 6\ 0\ 1} \end{array}$$

Rename Only!

$$\begin{array}{r} 6\ 7\ 3\ 0\ 0\ 4\ 6\ 0\ 0\ 9\ 8\ 0\ 0\ 6\ 1\ 0\ 0\ 2\ 7\ 8\ 0\ 0\ 0\ 5 \\ -\ \underline{4\ 1\ 4\ 7\ 0\ 6\ 5\ 9\ 0\ 4\ 7\ 0\ 0\ 1\ 0\ 0\ 0\ 3\ 8\ 6\ 0\ 5\ 0\ 1} \end{array}$$

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

Low Stress # 13

Name ____________________

Varied Renaming With Zeroes

Rename and subtract.

```
   4 8 0 9 6 0 0 4 5 3 1 2 0 0 6 0 7 8 0 0 4 5 6 0 0 0 0

 - 2 6 0 8 9 4 0 6 5 1 8 7 6 2 1 7 6 6 0 1 3 6 1 0 0 0 0
   -----------------------------------------------------
```

```
   9 6 0 0 0 7 6 5 1 3 2 4 0 6 0 8 9 4 6 7 1 0 0 0 7 0 0

 - 2 9 8 6 5 4 3 8 6 4 0 0 6 0 0 7 1 4 3 2 1 6 0 0 0 6 0 7
   -------------------------------------------------------
```

Low Stress # 14 Name ____________________

Varied Renaming With Zeroes

```
    8 4 5 2 0 0 0 6 0 0 3 0 0 6 0 0 0 0 7 0 0 6 0 4 3 0

  - 2 3 6 2 0 6 0 5 0 0 2 0 0 7 8 9 2 1 6 4 3 5 0 4 2 1
    ---------------------------------------------------
```

```
    7 6 3 4 0 0 0 6 0 2 3 0 6 2 7 0 4 0 3 2 1 0 1 0

  - 2 3 4 2 0 0 0 5 4 7 6 0 4 3 6 5 2 0 2 3 0 1 0 0
    -----------------------------------------------
```

```
    1 0 0 0 2 6 0 0 0 3 2 0 0 1 0 2 0 3 2 0 0 4 0 0 0

  - 1 0 0 0 1 3 6 2 7 2 3 0 1 2 0 1 2 0 1 0 9 6 8 0 2
    -------------------------------------------------
```

Low Stress # 15 Name ____________________

Varied Renaming With Zereos

  7 6 3 4 5 0 6 0 0 7 8 0 0 6 7 0 0 0 0 0 4 3 6 8

- 1 9 4 2 6 0 5 1 0 6 9 0 0 4 9 4 9 0 6 3 2 1 0 0

  8 6 4 2 0 0 0 0 9 0 0 0 0 0 0 4 9 8 0 4 2 1 0 0

- 3 6 9 4 0 8 6 1 4 0 2 3 0 0 0 1 8 9 0 3 2 1 0 0

  7 0 0 6 4 0 0 3 0 0 7 0 0 6 0 0 4 0 0 1 0 0 0 0

- 5 0 6 5 3 0 0 2 0 0 8 0 0 7 0 0 2 0 0 1 0 0 0 0

  7 9 0 0 6 0 0 2 0 0 4 0 0 0 6 0 0 0 1 0 0 6 0 0 7

- 2 7 0 1 5 0 0 1 0 0 6 1 2 4 5 0 0 1 0 0 0 5 0 1 0

BIBLIOGRAPHY

Afifi, A.A., & Azen, S.P. *Statistical analysis: A computer oriented approach* (2nd.ed). New York: Academic Press, 1979.

Ashlock, R.B. *Error patterns in computation: A semi-programmed approach* (2nd. ed.). Columbus, Ohio: Charles Merrill Publishing Co., 1976.

Athappilly, K.K. A meta-analysis of the effects of modern mathematics in comparison with traditional mathematics in American educational system (Doctoral dissertation, Western Michigan University, 1978). *Dissertation Abstracts International*, 1979, *40*, 6002A. No. 7908643.

Austin, G.R., & Prevost, F. Longitudinal evaluation of mathematical computation abilities of New Hamphire's eighth and tenth graders, 1963-1967, *Journal of Research in Mathematics Education*, 1972, *3*, 59-63.

Boyle, M. Effects of social reinforcement under simulated test and nontest conditions on the computational rate and accuracy of children using either the conventional algorithm or Hutchings' 'low stress' algorithm. (Doctoral dissertation, University of Maryland, 1973). *Dissertation Abstracts International*, 1975, *36*, 3499A. No. 75-78-734.

Bright, G. W. Assessing the development of computation skills. In M. N. Suydam, & R. E. Reys (Eds.), *Developing Computational Skills*. Reston, Virginia: The National Council of Teachers of Mathematics, Inc. 1978.

Brown, J.S., & Burton, R.B. Diagnostic models for procedural bugs in basic mathematical skills. *Cognitive Science*, 1978, *2*, 155-192.

Brown, J.S., & VanLehn, K. Towards a generative theory of "bugs." In T.P. Carpenter, J.M. Moser, & T.A. Romberg (Eds.), Addition and Subtraction: A Cognitive Perspective. Hillsdale, N.J.: Lawrence Erlbaum Associates, Publishers, 1982.

Brownell, W.A. An experiment on 'borrowing in third grade arithmetic'. Journal of Educational Research, 1947, 41, 161-171.

Brownell, W.A. & Moser, H.E. Meaningful vs. mechanical learning: A study in grade III subtraction. Duke University Studies in Education, 1949, 8, 1-207.

Case, R. A developmentally based theory and technology of instruction. Review of Educational Research, 1978,48, 439-469.

Case, R. General developmental influences on the acquisition of elementary concepts and algorithms in arithmetic. In T. Carpenter, J. Moser, & T. Romberg (Eds.), Addition and subtraction: A cognitive perspective. Hillsdale, N.J.:Lawerence Erlbaum Associates, Publishers, 1982.

Cochran, W.G. Analysis of covariance: Its nature and uses. Biometrics, 1957, 13, 261-281.

Collis, K.F. A study of concrete and formal operations in school mathematics: A Piagetian viewpoint. Hawthorn, Victoria: Australian Council of Educational Research, 1975.

Cox, L.S. Diagnosing and remediating systematic errors in addition and subtraction computation. Arithmetic Teacher, 1975, 22, 151-157.

Dashiell, W.H. An analysis of changes in affect and changes in both computational power and computational stamina occurring in mildly and moderately exceptional elementary school children after instruction in Hutchings' 'low stress' addition algorithm practice with unusually large examples, and exposure to one of two alternative performance options. (Doctoral dissertation, University of Maryland Arithmetic Center, 1974) Dissertation Abstracts International, 1975, 35, 7740A. No. 74-29-006.

Gagne, R. M. The conditions of learning (3rd. ed.). New York: Holt, Rinehardt and Winston, 1977.

Guilford, J.P. Some theoretical views of creativity. In H. Nelson, & W. Bevan (Eds.), Contemporary Approaches to Psychology, Princeton: Van Norstrand, 1967, 419-460.

Hamsin, J.M. Information-processing models for addition, subtraction and multiplication in mentally retarded and non-retarded. Toranto: American Association of Research in Education, 1978, ERIC Document Reproduction Service No. ED 168 226.

Hutchings, B. Low-stress subtraction. The Arithmetic Teacher, 1972, 22, 226-232.

Hill, S.A. Elementary school mathematics: More than the third R. Childhood Education, 1979, 55, 215-220.

Hutchings, B., Low-stress algorithms. In D. Nelson, & R.E. Reys (Eds.), Measurement in School Mathematics. Reston, Virginia: National Council of Mathematics, 1976.

Jungst, D. J Elementary mathematics methods: Laboratory Manual. Boston: Allyn and Bacon, Inc. ,1975.

Kalin, M., & Shami, M. Accelerating the learning rate of students with mild intellectual limitations in acquiring computational skills. Annapolis, Md.: Maryland State Department of Education Report, 1975.

Kane, P. New math as adolescent; interview. Teacher, 1975, 19, 20-22.

Keppel, J. Research design and analysis handbook. Englewood Cliffs, N.J.: Prentice-Hall, 1973.

Koppitz, E. Visual Aural Digit Span Test. N.Y.: Grune & Stratton Inc., 1977.

Lester, L. An investigation into the acceleration of basic skills of educationally disadvantaged rural primary children using Hutchings' low-stress procedures. (Doctoral dissertation, University of South Carolina, 1978) Dissertation Abstracts International, 1978, 39, 5908. No. 79-07613.

Myers, J.L. Fundamentals of experimental design. Boston: Allyn & Bacon, Inc., 1966.

Piaget, J. & Szeminska, A. The child's conception of numbers. New York: Humanities Press, 1952.

Reisman, F. K. Diagnostic teaching of elementary school mathematics: Methods and content. Chicago: Rand McNally College Pub. Co., 1977.

Resnick, L.B. Syntax and semantics in learning to subtract. In T.P. Carpenter, F. Moser, & T.A. Romberg (Eds.), Addition and subtraction: A cognitive perspective. Hillsdale, N.J.: Lawrence Erlbaum Associates, Publishers, 1982.

Romberg, T.A. An emerging paradigm for research on addition and subtraction skills. In T.P. Carpenter, F. Moser, & T.A. Romberg (Eds.), Addition and subtraction: A cognitive perspective. Hillsdale, N.J.: Lawrence Erlbaum Associates, Publishers, 1982.

Sherrill, J. M. Subtraction: Decomposition verses equal addends. Arithmetic Teacher, 1979, 26, 16-17.

Suydam, M.N., & Weaver, J.F. Addition and subtraction with show numbers. In Using research: A key to elementary school mathematics. Columbus, Ohio: ERIC Center for Mathematics and Science, 1975.

Thorndike, E. The psychology of arithmetic. New York: The Macmillan Company, 1922.

Webster, R.E. Short-term memory capacity in mathematics proficient and disabled students as a function of input modality-output modality. (Doctoral Dissertation, University of Connecticut, 1978). Dissertation Abstracts International, 1978, 39, 6665.
No. 7911419

Wilson, G.M. Teaching the new arithmetic. New York: McGraw-Hill, 1951.

Winer, B.J. Statistical principles in experimental design. N.Y.: McGraw-Hill, 1962.

BIOGRAPHICAL SKETCH

Daniel R. Paulson was born in St. Paul, Minnesota, on August 6, 1949, and graduated from Sibley High School, in West St. Paul, Minnesota, in 1967. He received a Bachelor of Science in Education from the University of Minnesota in 1971 with a major in history. In 1979 he was received the degree of Master of Education from the University of Central Florida with a major in specific learning disabilities. He has taught children with learning and behavior problems for eight years in elementary, junior high, and high schools. He also has two years experience as a program administrator for the Specific Learning Disabilities and Emotionally Handicapped Programs in Lake County, Florida.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Education.

Lynn Oberlin

Dr. Lynn Oberlin, Chairman,
Professor of General Teacher Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Education.

Michael C Hynes

Dr. Michael C. Hynes,
Professor of General Teacher Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Education.

Robert Algozzine

Dr. Robert Algozzine,
Professor of Special Education

This dissertation was submitted to the Graduate Faculty of the Division of Curriculum and Instruction in the College of Education and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Education.

August, 1983

Dean for Graduate Studies and Research